SOME REMARKS ON SYLLOGISTIC, DIALECTIC, AND THE STUDY OF THEIR HISTORY

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Abstract:

In this paper, I argue that one should approach the study of Ancient Logic, Greek, Medieval and Arabic through a better understanding of the relation between dialectic and syllogistic. No claims are made about these, however, except concerning Aristotle and the invention of syllogistic. Numerous defects of the early a-historical study of syllogistic by Łukasiewicz in terms of axiomatic systems are presented in section 2, that show its inappropriateness. It is further argued that a proper consideration of the context of Prior Analytics shows that it involved the common practice of dialectical games, a characterization of which, in terms of game semantics, is provided in section 3. In the concluding remarks, the interest of this new approach is illustrated by explaining aspects of Prior Analytics that had been hitherto left unexplained.

Résumé

Dans ce texte, j’argumente en faveur de la thèse selon laquelle l’étude de la logique ancienne, grecque, médiévale, et arabe doit être approchée à partir d’une meilleure interprétation des relations entre dialectique et syllogistique. Je ne dis cependant rien de ces logiques, et ne fais valoir des points qu’à propos d’Aristote. De nombreux défauts de l’approche anhistorique de Łukasiewicz, en termes de système axiomatique, sont présentés dans la section 2, pour en montrer le caractère inapproprié. Par la suite, je montre que la prise en compte du contexte des Analytiques premiers montre l’importance du rôle joué par la pratique répandue des joutes dialectiques, dont une caractérisation en termes de sémantique des jeux est proposée dans la section 3. La conclusion illustre l’intérêt de cette nouvelle approche en expliquant des aspects des Analytiques premiers laissés auparavant inexpliqués.

1 This paper is based on a lecture at a workshop on Arabic Logic in Medieval Philosophy, the Université de Kairouan, in April 2013. I would like to thank the organiser, Hamdi Mlika for his kind invitation. The paper is a sort of interim report on collaborative work with Benoît Castelnérac (Université de Sherbrooke), begun with (Castelnérac & Marion 2009), and refers to further work with Helge Rückert (Universität Mannheim) in (Marion & Rückert unpublished). I would like to thank both authors for their input, present in almost every paragraph, and especially Benoît Castelnérac, including also for his comments on an earlier version. In particular, the remarks of per impossibile syllogisms in Prior Analytics, in section 3 and the concluding remarks reflect our latest discussions. I use standard conventions for referring to Aristotle’s or Plato’s texts, and Robin Smith’s translation of Aristotle’s Prior Analytics, Oxford, Clarendon Press, 1989, as well as his translation of Topics, books A and Θ in Topics Books I and VIII, Oxford, Clarendon Press, 1997. Otherwise, translations from Aristotle are from J. Barnes (ed.), The Complete Works of Aristotle, The Revised Oxford Translation, Princeton NJ, Princeton University Press, 1984 and those from Plato are from J. M. Cooper (ed.), Plato. Complete Works, Indianapolis IN, Hackett, 1997.
The numerous figures [of the syllogism] resemble the flesh of a camel found on the summit of a mountain; the mountain is not easy to climb, nor the flesh plump enough to make it worth the hauling.

Ibn Taymiyya

1. On the Study of the History of Logic

In this paper, I wish to raise some issues concerning the study of the history of logic and its origin in Ancient Greece, in relation to what I take to be a radical misunderstanding of the role of dialectic, for which Aristotle wrote his *Topics* as a handbook (to which, as internal evidence shows, *On Sophistical Refutations* should be appended as a ninth and final book).\(^1\) The approach that I favour in the history of ideas is broadly derived from R. G. Collingwood and Quentin Skinner;\(^2\) its key idea can be stated in a very simple manner: in order to *understand* texts, e.g., those composing Aristotle’s *Organon*, it is better to see them as *acts* within a *context*. One acts within a given context in order intentionally to modify it, so the intention can be recovered from an understanding of the context and the change the act has provoked. A useful analogy here is with a particular move within a game of chess. One would hardly conclude that one has fully understood a given move merely when one has explained how it was done in accordance with the rules of chess; something more is expected, namely a reconstitution of the series of moves within which this move is embedded, so that one can understand the point of the move as, say, part of an overall strategy deployed by the player, even possibly as a key move, giving that player the upper hand and, say, leading to checkmate within a few turns. Likewise, a careful reconstruction of the context of a given text allows one better to understand its point – the author’s intention – in terms of interaction within a given context. For example, the text of Aristotle’s *Topics* clearly presupposes from its reader prior acquaintance with dialectical bouts, without which it is not clear what the point of the text might be: he does not even bother describing those bouts. Now the main claim of this paper is that one can reach a better understanding of syllogistic, as introduced by Aristotle in *Prior Analytics*, by placing it within the context the dialectical games of *Topics*.

On the other hand, lack of an interpretative context can lead to severe misunderstandings, e.g., in the case of poor Syriac and Arabic translations of the first book of Aristotle’s *Poetics* and a lack of knowledge of Greek tragedy – that is of Aristotle’s context – which led Averroes to think that Aristotle dealing not with ‘tragedy’ but with ‘eulogy’.\(^3\) The importance of dialectical games in the genesis of syllogistic, thus

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1. Aristotle speaks at the end of *On Sophistical Refutations* (see 34, 183b34f) of “the present inquiry”, but he clearly refers in the context to both *Topics* and *On Sophistical Refutations*. It seems only accidental that the latter was preserved in the *Organon* as an independent book.

2. For Collingwood, see (Collingwood 1978) and the ‘epilegomena’ to (Collingwood 1994) and, for Skinner, his papers collected in (Skinner 2002). As for the view, as interpreted here, see (Skinner 2001) and (Kobayashi & Marion 2011).

3. See, e.g., the famous statement in (Averroes 1986, 51), and, for a discussion (Leezenberg 2004). As the latter points out, it is not just the lack of knowledge of Greek tragedy which is at stake but the specific
its role in the origin of logic, has generally been disregarded, and this has to do partly with a poor understanding of Aristotle’s context, and partly with the specific, non-contextual approach adopted by historians of logic in the 20th century.

The study of the history of logic is, of necessity, linked with one’s understanding of the nature of logic. This may explain why this domain was in the doldrums until the revolution initiated by Frege at the end of the 19th century. As Heinrich Scholz put it, referring to Carl Prantl’s classic study, *Geschichte der Logik im Abendland*:

*When Prantl wrote his history of logic the type of modern formal logic which is now available in the shape of symbolic logic had not yet been called into being. There was, therefore, no dependable position by which such a history could be oriented and from which it could be surveyed. For, what formal logic really is we know only because symbolic logic provided the conceptual equipment needed to answer this problem.*

The development of logic after Frege thus naturally led to a renewal of the very discipline of the history of logic. Section 2, below, will be devoted to a critique of one of the early studies in that spirit, Jan Łukasiewicz’s influential *Aristotle’s Syllogistic from the Standpoint of Modern Formal Logic*. There is much to praise in the writings of early post-Fregean pioneers such as Łukasiewicz, but we are also bound to improve upon them, perhaps not in matters of detail, but at least in trying to make a fresh start. For this, one ought first to take a step back and make the following observations; in the above Collingwoodian spirit, they have first to do with the context of these early ‘modern’ historians of logic.

One might thus begin by noting that, early on, modern propositional and predicate logic had to supplant the dominant syllogistic inherited from the tradition. For that reason, as John Corcoran and Michael Scanlan once put it, writers tended in those days to “look upon Aristotle’s logic with jaundiced eyes”. Aristotle was rightfully chastised for claiming in *Posterior Analytics*, A 14, that demonstrations in “arithmetic, geometry and optics” are syllogisms of the first figure, e.g., of the mood *Barbara*. This is just false, as we all know, because syllogistic could not cope, *inter alia*, with the fundamental phenomenon of quantifier dependence, which occurs in as elementary a sentence as the arithmetical axiom that states that for all natural number \( x \), there is exists a natural number \( y \) which is the successor of \( x \). But Łukasiewicz went much further than this. He was, as far as I know, the first to claim that logic two different sources, in Aristotle’s syllogistic, as presented in his *Prior Analytics*, and in the beginnings of propositional logic

context of the Arabic tradition in which poetic syllogisms were conceived of as leading to conviction or assent.

1 (Prantl 1855-1867).
2 (Scholz 1961, vi).
3 See the influential (Łukasiewicz 1957), whose first edition appeared in 1951. Łukasiewicz had begun working on Aristotle in the 1930s, but his manuscripts were destroyed during the war. The earlier and rather similar (Miller 1938) appears to have had hardly any influence, while Łukasiewicz has been very influential. See, e.g., (Bochenski 1951) and (Patzig 1968).
4 (Corcoran & Scanlan 1982, 76).
5 The point is conceded in (Corcoran 1974a, 123). Other standard criticisms dating from the 19th century turned out not to be so convincing, e.g., the critique of Aristotelian existential import: although it is standardly held as a deficiency of Aristotelian syllogistic, hardly anyone observes that the same principles hold for our modern ‘many-sorted’ logics, as was noted by Timothy Smiley a long time ago (Smiley 1962).
in the writings of the Stoics.\footnote{Łukasiewicz 1967.} He also argued, however, that Aristotle’s syllogistic was a ‘logic of terms’ that could not stand on its own, so to speak, but presupposed a ‘propositional logic’, the existence of which Aristotle simply did not suspect. In other words, not only was Aristotle’s syllogistic was limited in scope, he did not realize that it needed, when fully expressed, something like our ‘modern’ logic.\footnote{If Łukasiewicz were right, how could Aristotle still deserve the title of ‘founder’ of logic? Corcoran has raise this point repeatedly. See (Corcoran 1974a, 98), (Corcoran 1974b, 280) and (Corcoran 1994).} The debates of yesteryears are not ours, we have to move beyond them, and I wish to take a closer look here at Łukasiewicz’s account of syllogistic, in order to provide, assuming that something went wrong along the way, a diagnosis.

What I have in mind is this: the revolution in logic brought about by Frege naturally led to a salutary renewal in the study of the history of logic but, by and large, studies in that field have tended to reflect a new set of prejudices rather than a stronger sense of historical sensitivity. Quite naturally, modern historians inherited from the tradition a narrow focus on Aristotle’s syllogistic, which they tended first to study from the axiomatic point of view first introduced in logic with Frege’s \textit{Begriffsschrift}, in 1879.\footnote{For Frege’s \textit{Begriffsschrift}, see (van Heijenoort 1967, 5-82).} There are two points to be made here. First, the particular angle from which Łukasiewicz studied Aristotle was not that of \textit{the} correct understanding of “what formal logic really is”, to use Scholz’s words. We now know that it simply derived from a particular philosophical understanding of logic, one among many, and that its key feature, the axiomatic model, is the source of many errors of interpretation. Secondly, the focus was on Aristotle’s syllogistic \textit{merely} because this was the legacy from the tradition, and that does not mean that there are no distortions involved in this legacy to begin with. Indeed, syllogistic had fossilized over the centuries, and after Frege, textbooks were still written that focussed solely on it; in Oxford, for example, H. W. B. Joseph’s \textit{Introduction to Logic} (1916), was still in use after World War II, while in France Jean Tricot’s \textit{Traité de logique formelle} (1930) was still reprinted in the 1970s.\footnote{See (Joseph 1916) and (Tricot 1973).} They remind one of Ibn Taymiyya’s words quoted above. This focus on syllogistic is, however, merely a bias inherited from the tradition, and, as I said, it was at first studied from the only (syntactic) standpoint available, that of the axiomatic system. More on this point in section 2.

Another prejudice inherited from the tradition is the idea that dialectic was \textit{superseded} by syllogistic. Sir David Ross expressed it in those terms:

\textit{We have neither the space nor the wish to follow Aristotle in his laborious exploration of the topoi, the pigeon-holes from which dialectical reasoning is to draw its arguments. The discussion belongs to a by-gone mode of thought; it is one of the last efforts of that movement of the Greek spirit towards a general culture, that attempts to discuss all manner of subjects without studying their appropriate first principles, which we know as the sophistic movement. What distinguishes Aristotle from the sophists, at any rate as they are depicted both by him and by Plato, is that his motive is to aid his bearers and readers not to win either gain or glory by a false appearance of wisdom, but to discuss questions as sensibly as they can be}
discussed without special knowledge. But he has himself shown a better way, the way of science; it is his own Analytics that have made his Topics out of date.¹

But it is also found in more than one study of the history of logic. For example, Father Bochenski could not even countenance that there could have been any sophisticated logical reasoning prior to Aristotle, because no logical rules had been explicitly formulated prior to the Stagirite. He thus found the reading of Plato’s dialogues “almost intolerable” because they contained “so many elementary blunders”,² and he also commented on Gorgias:

More [...] rules could be extracted from the great fragment of Gorgias; as, however, this text not only contains typically Stoic terms, but also betrays a very high level of logical skill unthinkable at that period, the principles used in it cannot safely be ascribed to the Sophist.³

Bochenski refers here to fragments of Gorgias’ treatise On Not-Being, handed down to us via Sextus Empiricus⁴ Against Logicians VII, 65f. and Pseudo-Aristotle’s On Melissus, Xenophanes and Gorgias, 979a12f., a treatise which actually contains a very intricate and a set of logically astute arguments.⁵ Bochenski could not bring himself, however, to grant that the argument as we known it was Gorgias’ true argument, and he uses the presence of typically Stoic vocabulary in Sextus to dismiss this possibility: the argument would, on his account, be from a Stoic and wrongfully attributed to Gorgias in both texts. His reason for this was simply that it it “unthinkable” that “very high level of logical skill” was possible during Gorgias’ time. Nonetheless, Gorgias’ lack of logical skill would be odd, given that he was a contemporary of Zeno as well as Plato, whose Parmenides contains, in its second part, the best preserved example of dialectic we have, with an unbroken stretch of almost thirty Stephanus pages, from 137c to 166c, comprising a series of 532 short questions with a yes/no answer by the proponent, forming a sequence of roughly 180 arguments, each ending in a contradiction or elenchus: the vast majority of these arguments are actually valid.⁶ And the logical sophistication and dialectic nature of Zeno’s arguments (about which more in section 3), can hardly be denied. One reason for Bochenski’s blindness to this will come up in section 2.

In essence, the view expressed by Ross in the passage quoted above is this: Aristotle wrote Topics as a handbook for dialectical games that were common practice in his days, and there are reasons to consider Topics an older text than Prior Analytics, such as the simple fact that the former is mentioned in the latter (for example at the very beginning, at A 1, 24¹12), therefore that syllogistic was meant to displace dialectic and rendered it otiose. It did not occur to him that this simply is a non sequitur. To explain its lack of relevance, even for the study of Aristotle, he associated dialectic instead with a “by-gone mode of thought”, the Sophistic movement. The latter has itself been terribly neglected

¹ (Ross 1949, 59).
² (Bochenski 1951, 17).
³ (Bochenski 1951, 17). For a similar passage, see (Bochenski 1961, 30).
⁴ See (Dillon & Gergel 2003, 67-76) for an accessible translation. For a detailed analysis, see (Castelnérac to appear).
⁵ As one can see, e.g., from the very detailed analysis in (Rickless 2007).
as Sophists were not even considered as philosophers properly so-called, and were not even included among ‘pre-Socratic’ philosophy. Given that, on the other hand, syllogistic plays a central role in the theory of scientific demonstrations in Posterior Analytics, these two treatises were the focus of much attention when dealing with the Organon, and Topics remains to this day one of Aristotle’s least studied treatises. This relative neglect also spread to early dialectic, from Zeno to Plato: it is perhaps not possible to ignore it altogether, but it is very seldom studied for its own sake.

One obvious difficulty with the idea that dialectic pertains to an era that ended when Aristotle discovered syllogistic is that they were also obviously played after him, e.g., by his adversaries, the Megarians, or in Plato’s Academy at least until Arcesilaus. Under one guise or another such as Quintillian’s altercatio or the obligationes of late Medieval philosophers, dialectic actually survived beyond Medieval ages at the Renaissance, and one is given no explanation for this awkward fact. If dialectic had been so closely related to the defunct world of the Sophists, why did its practice survived at least for close to two… millennia? The usual explanation, meant to minimize the importance of dialectic, involves a different argument: dialectic was merely practiced as a propaedeutic, i.e., as a sort of training, with no actual value for philosophy itself. Given that, somehow, one can do the philosophizing without the training, the need for dialectic as training was lost along the way. As we shall see in section 3, this claim is at least already very dubious for Aristotle himself, hence the importance of dialectic for his followers through the ages.

When 20th-century logicians, e.g., Nicholas Rescher, finally began to study in earnest the history of Arabic Logic, they naturally focussed on syllogistic within that tradition, neglecting again the commentaries of the Kitab al-Jadal—one of the names under which Topics was known—including by major philosophers such as Al-Farabi, Avicenna, and Averroes. They ignored as well the role that Jadal—the counterpart of dialectic within that tradition—also played within Islamic philosophy, theology and jurisprudence; the

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1 Although it has roots in Antiquity, the concept of ‘pre-Socratic’ philosophy, which implies a radical break between Socrates and his predecessors, is largely a construct of 19th-century German scholarship, under the deleterious influence of Hegel’s vision of the history of philosophy. See (Laks 2006). It is certainly of no help to understand the relation that Socrates and Plato had with their elders.

2 Here, a scientific ‘demonstration’ would be a particular kind of syllogism, whose premises are known to be true, necessary and universal, and which is explanatory of its conclusion, which is also true, necessary and universal.

3 The Megarians are also known for that reason as having formed the ‘dialectical school’. For an overview of the latter, see (Bobzien 2009). Diodorus Cronus’ arguments or the Sorites clearly have their origin in dialectic.

4 For an introduction, see (Brittain 2009).

5 See Quintillian’s altercatios praecepta in Institutio Oratoria, book vi, section iv. Quintillian’s altercatio were influential during the Renaissance, e.g., on Lorenzo Valla’s Dialectical Disputations (Valla 2012).

6 See (Spranzi 2011). It is worth mentioning here Ignacio Angelelli’s pioneering paper, which covers the modern period, (Angelelli 1970, 806-813).

7 The view that dialectic forms part of a philosopher’s training has, of course, its sources in Greek philosophy itself, but it is hardly clear that it was meant to be limited to this, i.e., that it was thought at the time that a philosopher could go about his business without the need first for dialectic exchanges that would bring out the arguments on both sides of an issue. This would go contrary to Aristotle’s practice as presented in section 3 below.

8 His (Rescher 1963) and (Rescher 1964) form an early sustained attempt at studying that tradition (but many of his conclusions have been superseded since). It is quite noticeable at all event that he never discussed dialectic and its Arabic counterpart, Jadal.
only thorough study to this day remains an unpublished Ph.D. thesis, dating from 1984, by Larry B. Miller. The topic is, as a matter of fact, hardly mentioned in surveys of Arabic logic. It is not my intention to contribute here to the study of any specific point within this very rich Arabic tradition, but merely to suggest that a better understanding of the relation between dialectic and syllogistic should be the foundation of future studies in this field too, not just in late Antiquity or Western Medieval philosophy.

To further this aim, I shall limit myself in what follows to two central points. First, I shall criticize the use of the axiomatic model for the study of the history of logic, focussing on the failings of Łukasiewicz’s book. This should provide us with a diagnosis of what went wrong and pave the way to a better approach to the study of Aristotle’s *Organon* (as well as the whole tradition of Western and Arabic Logic until at least the 13th and 14th century). Secondly, I shall offer a necessarily broad sketch of the dialectic that Aristotle wrote about, so that it can be refined and used as an object of comparison, for the purpose of clarification; one can thus plot the modifications of that practice over centuries and in different contexts. These issues cannot be discussed at the required level of scholarship, within the span of a few pages, so my intention is merely to provide an overall sketch, with all the unavoidable raccourcis, for which I apologize.

2. How not to Study the History of Logic

In this section, I would like to go over a number of problems with Łukasiewicz’s account of Aristotle’s syllogistic. There is little originality in the details, but I wish to orient my discussion towards a diagnosis that shows, first, that the conception of logic Łukasiewicz imposed on Aristotle’s *Prior Analytics* was inappropriate and, secondly, that a more profound, albeit rather obvious, failure of his account is precisely linked with his having neglected altogether to context of *Prior Analytics*. The two defects are in many ways interrelated.

(1) One of Frege’s innovations was to introduce, at the syntactical level, the axiomatic model in logic. The introduction of axiomatic thinking in logic was not a mere presentational device, however, as it also meant that one would look at logic not as a system of deductions of consequences from arbitrary, possibly false, premises, but as a system of proofs of logical truths based on logical axioms, couple with one or two inference rules, e.g., *Modus Ponens*. This view of logic as “the systematic study of the logical truths”, to quote W. V. Quine, influenced early historians of logic such as Łukasiewicz, who construed Aristotle’s syllogistic as an axiomatic system, in which syllogistic figures are not meant to be inference rules, but logical truths or ‘laws’ of the

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1 (Miller 1984).
2 See, e.g., the recent surveys by Tony Street, (Street 2004) and (Street 2008).
3 It is only recently that obligations have been studied from a point of view akin to that of this paper, in (Dutilh-Novaes 2007). See also the earlier work of (Stump 1989), (Yrjönsuuri 1993), and (Yrjönsuuri 1994) as well as (Uckelman 2012) for a recent overview.
4 See (Corcoran 1974a) and (Corcoran & Smiley 1982, 78).
5 (Quine 1986, vii).
6 For the full system, see (Łukasiewicz 1957, 88-90).
‘if..., then...’ form, with a conjunction of the two premises as the antecedent and the conclusion as the consequent.¹

Syllogisms were indeed traditionally understood as rules of inference composed of two premises and a conclusion, these being made up from four² basic forms of propositions, given here with a corresponding modern notation,³ derived from the traditional letters, $A, E, I,$ and $O$:

- **Universal affirmative:** ‘All $a$ are $b$’ $Aab$
- **Universal negative:** ‘No $a$ are $b$’ $Eab$
- **Particular affirmative:** ‘Some $a$ are $b$’ $Iab$
- **Particular negative:** ‘Not all $a$ are $b$’ $Oab$

A syllogism of the first figure, *Barbara*, would thus be, according to Łukasiewicz, a conditional:

If ‘All $a$ is $b$' and ‘All $b$ is $c$’, then ‘All $a$ is $c$'.

And thus of the form:

$$(Aab \land Abc) \rightarrow Aac$$

And not of the form of a rule of inference:

$$Aab \land Abc$$

$$Aac$$

To read rules of inference as ‘laws’ of the form of conditionals is also the mistake committed by Bochenski,⁵ and it explains his inability to cope with any logical fragment prior to Aristotle, which we saw above. The view here seems to be that use of a ‘law’ cannot properly be attributed to someone unless it has already been made explicit. Such a

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¹ See (Łukasiewicz 1957, 1-3, 10, 14-15, 20-21) and, following him, (Patzig 1968, 3-4). This claim, and Łukasiewicz’s exegetical grounds for it, was challenged by J. L. Austin in his review of the first edition of Łukasiewicz’s book (Austin 1952, 397-398). Austin’s assessment was shared by (Prior 1955, 116) and (Rose 1968, 24-26). Since (Smiley 1973) and (Corcoran 1974a) the balance has definitely tipped against Łukasiewicz on this point. For obvious, convincing arguments against Łukasiewicz and Patzig, that rely on no particular exegetical point, see (Lear 1980, 9-10, n. 17).

² I leave aside here the fifth form of the indefinite ‘$a$ is $b$’, which does not appear in inference rules for syllogisms.

³ This is the notation in (Smiley 1973), (Corcoran 1974a) and (Lear 1980). Robin Smith has popularized a slight variant, e.g., in (Smith 1989a, xixf) and (Smith 1998, 34).

⁴ This is not Aristotle’s own form of expression. For $Aab, Eab, Iab,$ and $Oab,$ he would say, respectively, ‘$b$ belongs to all $a$’, ‘$b$ belongs to no $a$’, ‘$b$ belongs to some $a$’, and ‘$b$ not-belongs to some $a$’. He even had another form of expression, e.g., at *Prior Analytics*, A 4, 26a, quoted below: ‘to be predicated of all’, ‘to be predicated of none’, and so forth.

⁵ (Bochenski 1951, 3-4).
view is open to an objection derived from Lewis Carroll’s paradox of inference,¹ and it is preferable simply to assume that one can act according to a rule without following an explicit statement of it, i.e., that Gorgias and Plato were able to display considerable logical skills before any logical law they might have used was made explicit.²

(2) The above basic forms of propositions were construed by Łukasiewicz as containing non-logical constants that represent relations between universal terms. These are relations between terms, not propositions, so, according to Łukasiewicz, Aristotle’s contribution was thus limited to the development of a ‘logic of terms’, where A would express inclusion, E exclusion, I overlap, and O non-inclusion. But the above formula uses the connectives ‘&’ and ‘→’ that link propositions, so Aristotle’s presumed axiomatic system presupposes a propositional logic as ‘underlying logic’.³ Aristotle was thus faulted by Łukasiewicz for having ignored this.⁴

(3) Likewise with quantifiers – another of Frege’s key innovations in his Begriffsschrift. One must note that Łukasiewicz treated quantifier expressions ‘all’, ‘no’, ‘some’, ‘not all’ simply as part of his constants, A, E, O, and I, that represent relations between terms. While we would recognize today the importance of Aristotle’s contribution to the study of these expressions, Łukasiewicz was just dismissive and claimed that Aristotle “had no clear idea of quantifiers”.⁵ He believed that Aristotle connected instead quantifiers with ‘syllogistic necessity’, i.e., Aristotle’s claim that the conclusion of a syllogism must follow from the premises, e.g., at Prior Analytics, A 3, 26a:

**if A is predicated of every B and of every C, it is necessary for A to be predicated of every C.**

Łukasiewicz’s conditionals, such as the one above, are thus generalized with respect to their schematic letters, and would rather look like:⁷

\[ \forall a \forall b \forall c \ (Aab \& Abc) \rightarrow Aac \]

Again, these quantifiers would thus form part of an underlying logic whose existence was, according to Łukasiewicz, not suspected by Aristotle.⁸ I shall say more about this completely wrongheaded view in my concluding remarks.

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¹ See (Marion to appear).
² The view expressed here is a form of ‘inferentialism’ inspired partly by Robert Brandom’s. See (Marion 2010) and (Marion 2011).
³ The notion of ‘underlying logic’ comes from (Church 1956, 57-58 & 317).
⁴ (Łukasiewicz 1957, 14-15, 48-49).
⁵ (Łukasiewicz 1957, 83).
⁶ Łukasiewicz also believed that Aristotle also involved quantifiers with the notion of proof by ekthesis or instantiation (Łukasiewicz 1957, 84). But Łukasiewicz’s construal of proofs by ekthesis is also deficient. See footnote 56 below.
⁷ This universal conditional is Łukasiewicz’s axiom 3, (Łukasiewicz 1957, 88).
⁸ Since quantifiers are involved, one ought not to speak with Łukasiewicz of a ‘propositional logic’. Corcoran notes (Corcoran 1974a, 95) that Łukasiewicz’s rule of substitution at (Łukasiewicz 1957, 88) implies that he expressed universal quantifiers by means of free variables, so that the underlying logic allegedly presupposed by Aristotle would rather be a ‘free variable’ logic.
(4) There were no axioms in Prior Analytics. In order to set up an axiomatic system, Łukasiewicz decreed that the syllogistic ‘principles’ corresponding to the syllogisms Barbara and Datisi would be axioms, along with the ‘laws of identity’, as he calls them, ‘Aaa’ and ‘Iaa’. Łukasiewicz needed these to get his system going, so to speak, but there is obviously no textual basis in Aristotle, so the artificiality of Łukasiewicz’s system should be patent. It should thus be clear by now that his reliance on the axiomatic model and the concomitant view of logic as the systematic study of the logical truths led him to propose a model for Aristotle’s syllogistic that hardly fits it. It is worth digressing briefly from the list of defects to point out the obvious, namely that this standpoint is not the sole one available; reasons ought to be given by those inclined to use it today for its superiority as an investigative tool. Gerhard Gentzen introduced axiomless systems of logic that contain only rules of inference in 1934,\(^1\) the virtues of which were, alas, not generally recognized until the 1960s.\(^2\) Although the two modes of presentation, ‘Frege-style’ and ‘Gentzen-style’, are equivalent, it is at least arguable that the former is at the source of some philosophical difficulties,\(^3\) or simply that it is not suited at least for some purposes. There is certainly no need to see axiomatic systems everywhere when one looks at the history of logic, given that the introduction of that approach only dates from Frege. Moreover, that these are less suited for modelling Aristotle’s syllogistic has been independently shown by John Corcoran\(^4\) and Timothy Smiley\(^5\) in the early 1970s, when they interpreted Aristotle’s syllogistic not as an axiomatic system requiring an underlying logic, but as an underlying logic itself,\(^6\) which is best modelled (in the ordinary sense of the word ‘model’)\(^7\) as a Gentzen-style system. They also gave completeness proofs for their respective systems and thus restored Aristotle’s stature as a logician. Exegetical matters cannot be addressed here, but it is worth noticing that the superiority of this approach has been recognized since by Robin Smith:

> One principal virtue of Corcoran’s approach […] is that it permits a formal model which stays very close to Aristotle’s actual text, since it allows us to read formally precise natural deductions straight out of it.\(^8\)

This remark coupled with Łukasiewicz’s own admission that his reconstruction of Aristotle’s syllogistic relies on notions, e.g., propositional logic, that were introduced after Aristotle,\(^9\) should be sufficient to rule it out as an adequate interpretation of Aristotle, irrespective of the fact that it is inadequate in many other respects, e.g., in its treatment of the notion of proof by ekthesis.\(^10\)

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\(^1\) See (Gentzen 1969).
\(^2\) The turning point was a monograph by Dag Prawitz now re-edited as (Prawitz 2006).
\(^3\) See also (Dummett 1981, 432-435).
\(^4\) See (Corcoran 1972, 1973, 1974a, b), as well as (Corcoran 1994).
\(^5\) See (Smiley 1962, 1973, 1994). See also (Lear 1980).
\(^6\) See, e.g., (Corcoran 1974a, 87-88).
\(^7\) See footnote 1 in (Corcoran 1974a, 124-125). Corcoran’s model is his system D, (Corcoran 1974a, 108-110).
\(^8\) (Smith 1989, xvii).
\(^9\) Again, (Łukasiewicz 1957, 48-49). See also (Łukasiewicz 1968).
\(^10\) See (Lear 1980, 4), (Smith 1982), and (Smith 1983).
(5) More remarkable, however, is the fact that Łukasiewicz could not handle at all Aristotle’s meta-theoretical claims. It has often been noted that he could not account for the central claim at Prior Analytics, A 7, 29b1-2 that “it is possible to reduce all deductions to the universal deductions in the first figure”. Łukasiewicz does not even attempt to show this, while Corcoran succeeds in explaining how Aristotle proceeds. According to Smiley, “the price of accepting Łukasiewicz’s account of syllogisms is his wholesale rejection of Aristotle’s account of their reduction”; a major indictment.

(6) Aristotle had, however, another meta-theoretical claim, which is seldom discussed in the secondary literature, but rather important in the context of this paper, concerning direct or ‘probative’ syllogisms, such as those of the mood Barbara, and per impossibile syllogisms, i.e., syllogisms where one supposes the contradictory of what one wishes to prove and then derives a conclusion which is obviously false. For example: Suppose that that ‘Not all b are d’, then since ‘All b are c’, it follows that ‘Not all c are d’, but ‘All c are d’, therefore ‘All b are d’. One can extract from this per impossibile syllogism the universal affirmative propositions to form a direct, probative syllogism of the first figure:

\[
\begin{align*}
&\text{\textit{Abc Acd}} \\
&\text{\textit{Abd}}
\end{align*}
\]

Now, Aristotle makes the remarkable claim that one can always recover a direct syllogism from per impossibile one and vice-versa.

\begin{quote}
For whatever is proved probatively can also be deduced through an impossibility by means of the same terms, and whatever is proved through impossibility can also be deduced probatively.
\end{quote}

\begin{quote}
Everything concluded probatively can be proved through an impossibility, and whatever is proved through an impossibility concluded probatively, through the same terms.
\end{quote}

But Łukasiewicz could not even begin to provide an account of this claim, given that he treated syllogisms as conditionals, not deductions. He thus faulted Aristotle once more, this time for failing to see the obvious, namely that one does not negate a conditional by negating its consequent. He then went on to show Aristotle how to proceed, using a single propositional law, which would, again, be part of the underlying logic that Aristotle had presupposed. If, however, one is ready to entertain the idea that per impossibile syllogisms are dialectical, i.e., the very arguments found in dialectical games in the

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\footnotesize
1 Łukasiewicz 1957, 45. Corcoran does this within his system D (Corcoran 1974a, 113-115).
4 This example is taken from (Lear 1980, 9).
5 Respectively, Prior Analytics A 29, 45b26-28 and B 14, 62b38-40. The claim also occurs at B 14, 63b12-21.
6 Łukasiewicz 1957, 56.
\end{flushright}
Academy and elsewhere,¹ this meta-theoretical claim becomes crucial. To see why, I must first talk about dialectical games in the next section.

(7) Alongside a syntactical approach to logic in terms of axiomatic systems, there are also semantic approaches such as the well known ‘model-theoretical semantics’, based on a conception of meaning in terms of ‘denotation’, i.e., of some entity attached to the expression. It It would be, however, perfectly anachronistic to ascribe to Aristotle a Tarskian notion of logical consequence. Although this is not yet the conception that informs Łukasiewicz, one can add another reason to think that the larger tradition in which it belongs to be inappropriate for the study of Aristotle. It would require that one conflates ‘syntactic’ and ‘uninterpreted’, and Aristotle’s use of schematic letters does not imply that he had a semantic conception of consequence. As Jonathan Lear pointed out, he was

… working with the presemantic idea of interpretation by replacement: a statement-form is seen to have various instances. One obtains an interpretation of a syllogistic formula by substituting specific terms, of the appropriate logical category, for the schematic letters. Every syllogistic inference is valid under replacement in that for every substitution of terms which makes the premises true, the conclusion is true.²

Moreover, one would then need, under the model-theoretical semantics conception, to provide a semantic analysis of the notion of consequence that the syntactic relation was meant to capture and it is clear that Aristotle did not see the need do this. Recall the notion of ‘syllogistic necessity’ at Prior Analytics, A 3, 26a, quoted above or the very definition of ‘syllogism’, translated here as ‘deduction’ at A 1, 24¹18-20:

A deduction is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity, because the things are so.

Aristotle never defines this notion of ‘resulting’ or ‘following of necessity’, he merely presents moods of the first figure as ‘perfect’ or ‘complete’ syllogisms, because in their case nothing needs to be added “for the necessity to be evident” (A 1, 24¹22-25). It would be interesting to relate this approach, based on an intuitive recognition of the validity of some basic cases (to which other forms can be reduced), to John Etchemendy’s notorious critique of the model-theoretical of logical consequence as being unable to capture the intuitive notion of validity of an inference.³ By comparison, the approach favoured Corcoran and Smiley is based on a conception of meaning as use, where the meaning of the logical particles is given in terms of rules that determine how these particles are used in proof. This is why it is often called a ‘proof-theoretical semantics’. This approach is already more appropriate to the study of Aristotle with respect to this last point.

¹ This has been proposed in (Shorey 1889, 462) and (Lear 1980, 39).
² (Lear 1980, 8). For more on the intuitionist notion of interpretation by replacement, see (Dummett 1977, 218f.)
³ See (Etchemendy 1990).
3. The Context: Dialectic

Reliance at the syntactic level on the axiomatic model is only part of the reasons why Łukasiewicz went wrong; one should also notice, in accordance with the above-mentioned Collingwoodian perspective, a remarkable lack of sensitivity to the historical context. Indeed, Łukasiewicz did not attempt at all to understand Aristotle’s Prior Analytics within the Stagirite’s context, but rather merely to give an axiomatic model, faulting the latter at every step, when the model did not fit. The main claim of this paper is that the context within which one should try and understand the emergence of syllogistic is that of the practice of dialectical games. So far, we have seen that the switch to Gentzen-style systems of natural deduction, with Corcoran and Smiley led to a major improvement, in part by avoiding (1)-(7) above, but I would like to propose yet another change of perspective for the study of dialectic. After all, taking a wider perspective, one may also blame, the likes of Corcoran, Smiley, or Lear for having produced works that are a-historical, albeit to a lesser extent because of their greater degree of fit with the text.

There exists another approach based on a conception of meaning as use, where it is given in terms of rules that determine how these particles can be used in certain (language) games, hence the name ‘game’ or ‘interaction’ semantics.¹ In the 20th century, Paul Lorenzen was the first to introduce a game semantics, under the name of ‘dialogical logic’, in ‘Logik und Agon’ (1960).² His ideas were immediately developed by Kuno Lorenz, who introduced vocabulary and ideas from game theory.³ Later on, Jaakko Hintikka also developed independently a ‘game-theoretical semantics’ on a different, model-theoretical basis.⁴ For that reason, and because of the presence of an equivalent to the Socratic Rule, below, the dialogical games (Dialogspiele) of Lorenzen and Lorenz is to be favoured, about which I need to say a few words first.⁵

In terms of game theory, dialogical logic is the idea of defining logical particles in terms of rules for non-collaborative, zero-sum games between two persons, the proponent P of the initial thesis, and an opponent O, against whose attacks P defends the thesis, and to define truth in terms of the existence of a winning strategy for P. These games are defined by two sets of rules. There are structural rules that regulate the general course of the games, such as these:

- The initial formula is asserted by P. (Starting rule)
- P and O move alternately.
- Each move following the initial one is either an attack or a defence.
- One of the players wins if and only if it is the other’s turn but he cannot move

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¹ The expression ‘semantics of interaction’ has the advantage of avoiding some of the inappropriate connotations linked with the concept of ‘game’; it was only introduced recently in (Keiff & Rahman 2010).
² (Lorenzen 1960).
³ The original papers by Lorenzen and Lorenz are collected in (Lorenzen & Lorenz 1978).
⁴ Hintikka’s original proposal is reprinted as Chapter 3 of (Hintikka 1973).
⁵ The following presentation is closely modelled on that in sections 4 and 5 of (Marion & Rückert unpublished). For more on dialogical logic, see (Rückert 2001) and (Keiff 2009).
(either attack or defend). (Winning Rule)\(^1\)

There are two kinds of moves, attacks and defences, therefore two roles, ‘attacker’ and ‘defender’. (These will become, later on, the roles of ‘asking questions’ and ‘answering questions’.) This distinction does not coincide with the distinction between the players, who are named \(P\) and \(O\) depending on who puts forward at the beginning the thesis on which the game will be played.\(^2\)

In addition, there are also rules for the logical particles. Stated informally these rules are as follows:

- When one of the players puts forward ‘\(A \& B\)’, it is assumed that he would be able to defend both conjuncts, so the other player’s attack will consist in choosing one of the conjuncts, and the first player has then to defend it.\(^3\)

- Similarly, when a player puts forward ‘\(A \vee B\)’, he is assumed to be able to defend at least one of the disjuncts, and he is asked to choose one, in order to defend it.

- If one of the players puts forward an implication ‘\(A \rightarrow B\)’, the other player can attack it by conceding \(A\) in order to force him to defend \(B\). This corresponds to the intuitive meaning of a conditional, but, having put forward \(A\), the other player is in turn liable to be attacked himself.

- Finally, when a player puts forward a negation ‘\(\neg A\)’, the other player has no choice but challenge this by putting forward \(A\), the roles (attack vs. defence) are then exchanged, as it is the other player that has now to defend \(A\).

- The rules for the quantifiers are as follows: when a player puts forward ‘\(\forall x \ A[x]\)’,\(^4\) the other player chooses a constant \(a\) in order to force him to defend that \(A[x/a]\).\(^5\) Finally, when a player puts forward ‘\(\exists x \ A[x]\)’, the other player can only attack by asking him to choose a constant \(a\) and defend that \(A[x/a]\).

These particle rules may be said to form, when taken along with a specific set of structural rules, a semantics, in the sense that they provide an explanation of the meaning of the logical particles.

The following point needs to be explained carefully, because it will turn out to be highly relevant the understanding of dialectic. In a dialogical game, complex formulas are decomposed into atomic formulas, so that the meaning of complex formulas depends on the meaning of the atoms. The point of dialogical logic is that, instead of presupposing

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\(^{1}\) There are rules that allow one to define classical and several non-classical logics, but these are not relevant here. See (Rahman & Keiff 2005) and (Rückert 2011).

\(^{2}\) As the rules for negation and the conditional make plain, roles can be exchanged several times in the course of a dialogue, so that \(P\) might become the attacker and revert to being the defender more than once (and vice-versa for the opponent).

\(^{3}\) The use of the masculine in this paper is only meant to reflect on the fact that, historically, it was men who played dialectical games.

\(^{4}\) Where \(x\) may occur freely in \(A\).

\(^{5}\) Where the occurrences of \(x\) in \(A\) have been replaced by \(a\).
anything about the meaning of the atomic formulas, one asks what kind of strategy \( P \) could use given that \( P \) has no knowledge of the meaning of the atomic formulas. Under these conditions, \( P \) has only one strategy, namely what is now commonly named the ‘copy cat’ strategy in computer science, i.e., relying only on information that the other player, \( O \), has already introduced into the game. This key strategic thought motivates the introduction of the distinctive feature of dialogical logic, which forms no part of the semantics, the Formal Rule:\(^1\)

- \( P \) may not introduce atomic formulas, any atomic formula must be put forward by \( O \) first. (Formal Rule)\(^2\)

This rule is crucial because it makes the plays independent of the meaning of the atoms involved in the complex formulas. With it one can define formal validity: A formula is valid in a given dialogical system if and only if \( P \) has a formal winning strategy for this formula, i.e., \( P \) has, for any choice of moves by \( O \), at least one possible move at his disposal.

One could invert the roles of \( P \) and \( O \) in the Formal Rule, so that \( O \) is not allowed to introduce atomic formulas unless \( P \) has put them forward first. The logically false or invalid formulas would then be those for which there exists a winning strategy for \( O \). The Formal Rule is distinctive of dialogical logic and it may be better suited than its relatives for the study of Greek dialectic for the simple reason that it includes also a similar, albeit slightly different rule, to be called below the ‘Socratic Rule’ for dialectical games.\(^3\)

Although Lorenzen alluded to dialectic in ‘Logik und Agon’, until very recently practically nobody followed his hint and looked for a set of rules for ‘dialectical games’ using dialogical logic as a tool.\(^4\) Plato’s dialogues can be seen in this context as providing us with numerous illustrations of dialectical exchanges. They usually involve the proponent \( P \) of a given thesis, say, \( A \), who answers questions from an opponent \( O \), usually Socrates, and the dialogue begins with Socrates eliciting from his adversary the initial assertion \( A \), often as an answer to questions of the form ‘What is \( X \)?’. As \( O \), Socrates then tries to show that \( A \) is inconsistent with other assertions that \( P \) also happens to believe. To do so, he has first to elicit assent from \( P \) to further assertions, say, \( B_1, B_2, \ldots, B_n \), through a series of questions, the role of \( P \) being ideally reduced to yes/no answers. Taken together, these assertions form a set \( \{ A, B_1, B_2, \ldots, B_n \} \), which may be called \( P \)’s ‘scoreboard’. Then Socrates as \( O \) logically infers an inconsistency from this and gets \( P \) to recognize that he contradicted himself. Given that one might as well speak of

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\(^1\) This Formal Rule should therefore not be counted among the structural rules that define the semantics together with the particle rules. Alas, it has nearly always been listed among these in previous presentations of dialogical logic.

\(^2\) A more complete statement of the Formal Rule would include, for an obvious reason, that atomic formulas cannot be attacked.

\(^3\) In (Castelnérac & Marion 2009), this rule was simply called the ‘Formal Rule’ for dialectical games, but this might lead to confusion.

\(^4\) (Castelnérac & Marion 2009) and (Marion & Rückert unpublished). For previous ideas in that direction see (Ebbinghaus 1964, 48 & 57-58 n.1) and (Krabbe 2006, 666-670).
an inference to ‘impossibility’ (*adunaton*), this inference can be expressed formally, with our symbol ‘⊥’ for ‘absurdity’, as:

\[ A, B_1, B_2, \ldots, B_n \vdash \bot \]

This is the *elenchus*, which brings the exchange to a close.

Given the importance of dialectic as a method, it would be expected that handbooks were written to teach one how to play them, but only Aristotle's *Topics* have survived to this day. Diogenes Laertius mentions in *Lives of Eminent Philosophers*, XIII, 55, a *Techne Eristikon* and two books of ‘antilogies’ or contradictory speeches among works by Protagoras, and Aristotle mentions towards the end of *Sophistical Refutations* (34, 183b, 37), the previous practice, to which he associates the name of Gorgias, of handing out to students speeches in the form of questions and answers to be learned by heart. All of these are lost but a fragment called *Dissoi Logoi*, consisting of opposite speeches on a given thesis, survived attached to Sextus Empiricus.¹ Plato’s own dialogues may also have had a didactic purpose, e.g., *Euthydemus* could be used to teach fallacies.

Plato’s dialogues also contain many ‘meta-discussions’, when participants break off the exchange to reflect on the way they proceed, the content of which can be taken, in conjunction with Aristotle’s *Topics*, Books Α and Θ, to try and devise a set of rules for dialectic. These would include the following structural rules:²

1. Games always involve two players: a proponent *P* and an opponent *O*.
2. A play begins with *O* eliciting from *P* his commitment to an assertion or thesis *A*.
3. The play then proceeds through a series of alternate questions and answers. *O* asks questions such that *P* may give a ‘short answer’, ideally ‘yes’ or ‘no’ (*Topics, Θ 2, 158a 15*).
4. Proceeding thus, *O* elicits further commitments from *P*, e.g., commitment to assertions *B_1, B_2, …, B_n*, which can be conceived as added to *P*’s ‘scoreboard’.
5. *O* may not introduce any thesis, *P* must commit himself to any thesis used by *O*. (Socratic Rule for Dialectical Games)
6. Having elicited from *P* commitment to, say, *B_1, B_2, …*, and *B_n*, *O* can then ‘syllogize’, i.e., ‘take together’ or ‘add up’,³ and infer to impossibility from *A, B_1, B_2, …, B_n* The result, again, is the *elenchus*:

\[ A, B_1, B_2, \ldots, B_n \vdash \bot \]

7. If *O* has driven *P* into an *elenchus*, the play ends with *O* winning. *P* wins by avoiding being driven into an *elenchus*. (Winning rule)

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¹ See (Dillon & Gergel, 317-333).
² For the full set of rules, see (Castelnérac & Marion 2009). I skip some rules of lesser importance within the context of this paper. A revised discussion of the full set of rules will come in a projected book to be co-authored with Benoît Castelnérac.
³ As in Socrates saying: “Join me, then, in adding up what follows for us from our agreements” in *Gorgias* 498c.
Consideration of these rules already shows some differences between dialectical games and the modern games of dialogical logic. Since the purpose of dialectic is to test the consistency of $P$'s beliefs, Rule 5 – the Socratic Rule – is of fundamental importance. In dialogical logic, the player whose moves are thus restricted is $P$, while in dialectical games, it is $O$. This reflects the fact that in dialectic it is $O$'s goal to argue for inconsistency on $P$'s part.\footnote{The Socratic Rule is very similar to the Formal Rule insofar as, in both cases, one player is not allowed to introduce certain assertions by himself: in dialogical logic that applies only to atoms, while in dialectical games it applies to any premise. The purpose of the Formal Rule is to make the plays independent of the meaning of the non-logical expressions. But in dialectical games the players use an interpreted language.} Plato was very lucid concerning this key point, as the Socratic Rule is given a clear motivation within his dialogues in terms Socrates’ ‘avowals of ignorance’ – given that Socrates is playing as $O$ – as well as the ‘doxastic’ or ‘say what you believe’ constraint on $P$'s answers, e.g., at Protagoras 333c or Charmides 166d-e.\footnote{For more details about the ‘say what you believe’ constraint and Socrates’ avowals of ignorance, see (Castelnérac & Marion 2009, 57-59 & 61-62).} Indeed, it is of the utmost importance for Socrates qua $O$ that he does not introduce a premise of his own in $P$’s scoreboard, if he is convincingly to infer contradiction on $P$’s part. Otherwise, one would simply counter the charge of inconsistency by pointing out that one had not agreed to this or that premise. It is therefore important that the premises are put in $P$’s scoreboard only once $P$ has granted them – this is the ‘say what you believe’ constraint – but also that Socrates insists on his having no view on any given matter during the exchange – this being the ‘avowal of ignorance’, e.g., in the middle of the game in Lesser Hippias 372b-e. As it turns out, Socrates very often introduces premises, but he always requests assent from the respondent.

It is important now to distinguish these dialectical games from their use as a method of inquiry. The latter was presented by Plato in a crucial methodological passage at Parmenides 135d-136c. At 135c-136b, an old Parmenides is teaching young Socrates:

$$\text{[...]} \text{if you want to be trained more thoroughly, you must not only hypothesize, if each thing is, and examine the consequences of that hypothesis; you must also hypothesize, if that same thing is not.}$$

The method of enquiry suggested here involves, on a given topic, not only testing $A$ but also its negation, $\neg A$. Aristotle also describes dialectical games at Topics, Θ, 2, 158b14-25 as starting with the proponent being asked to choose an answer to a question of the form ‘Is it $A$ or $\neg A$?’. He usually speaks in this context of *aporia* or ‘difficulties’, ‘puzzles’, and he points out in Book Z that these emerge precisely when, ‘going through the difficulties on either side’, the results are equally puzzling:

Likewise also an equality between contrary reasonings would seem to be a cause of perplexity; for it is when we reflect on both sides of a question and find everything alike to be in keeping with either course that we are [puzzled] about which one we are to do. (Topics, Z, 6, 145b17-20; translation slightly modified.)

What he called ‘going through the difficulties on either side’ involves testing dialectically both answers, $A$ and $\neg A$. This may be called the ‘method of dialectical games’.
One could argue that this is also the form exhibited by Gorgias’ arguments in his treatise *On Non-Being*, mentioned above, but, more importantly, one should note further that both Plato, at *Parmenides* 128b-e and 135e, and Aristotle, as reported by Sextus Empiricus and by Diogenes Laertius, attributed this method of enquiry to Zeno. It could indeed be argued that Zeno’s four arguments about motion – the only set of surviving arguments extensive enough for us to draw some conclusions concerning his method – are of that form. Aristotle himself frequently alludes to these arguments as being ‘against motion’, a frequent misunderstanding. It is not even clear that they can be grouped together because they deal primarily with motion. Under a reading backed by Plato in the opening pages of his *Parmenides* (128c-d), and revived at least since Paul Tannery’s seminal paper, and they are meant to defend Parmenides’ monism by deriving contradictions from the contrary hypothesis, that ‘there are many’ (*ei polla esti*), including that it is incompatible with the existence of motion. The arguments would run roughly like this. Zeno would get his adversary first to concede that there is motion, then, supposing that ‘there are many’, that space is divisible. Then he would raise the ‘Is it *A* or ¬*A*?’ question: Is it infinitely divisible or not? If the adversary chooses that it is only infinitely divisible, then by ‘The Stadium’, Zeno would get his adversary to concede, at least under the version at *Physics* Z. 9, 239*11*, that a runner could not reach the other end of a stadium because he would have to traverse the first half, and then the first half of the first half, and so forth ad infinitum. If, however, the adversary chooses instead that space is *not* infinitely divisible, then by ‘The Arrow’ at *Physics* Z. 9, 239*5-7*, Zeno derives the claim that an arrow must be motionless in an instant, as indivisible *minima*, because if it were to change position, the instant would be divisible; so the arrow is always at rest.

Thus one needs two arguments, one for *A* and one for ¬*A*. But the ‘paradoxes’ come in pairs: ‘The Moving Rows’ is, like ‘The Arrow’, also an argument deriving a contradiction from the claim that ‘space and time consist of indivisible minima’, while ‘The Achilles’ complements the ‘The Stadium’ for the opposite assumption. Why four arguments? The answer is again in the methodological passage of *Parmenides*, where Parmenides tells to the young Socrates at 136a-b:

“*take as an example this hypothesis that Zeno entertained: if many are, what the consequences must be for the many themselves in relation to themselves and in relation to the one, and for the one in relation to itself and in relation to the many? And, in turn, on the hypothesis, if many are not, you must again examine what the consequences will be both for the one and for the many in relation to themselves and in relation to each other.*”

Using this as a cue, we can see that in each pair, one argument derives a contradiction from a body’s motion considered *in itself*, these are ‘The Arrow’ and ‘The Stadium’, while

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2 For example, in *Prior Analytics* B 17, 65*18-19* or *Topics*, Θ, 8 160*8-9*.
3 (Tannery 1885).
4 The version reported at *Physics*, Θ 8, 263*15-18*, 263*3-9* has it that a runner could not reach the other end of a stadium because he would have to traverse an infinity of points in a finite time, which is impossible, although a runner can reach the other end of a stadium, although a runner can reach the other end of a stadium.
the other argument, ‘The Achilles’ and ‘The Moving Rows’, derive a contradiction from a body’s motion in relation to the movement of another body. So the arguments would be structured like this, around divisibility, not motion.  

![Diagram showing the relationship between space and time and motion](image)

One could also argue that this is also the form taken by the eight series of deductions in the second half of *Parmenides* (137c-166c). As was said above, it contains 532 short questions with a yes/no answer by the proponent, forming a sequence of roughly 180 arguments. These are clearly organized forming eight series of deductions. Indeed, in the very same fashion, the arguments are structured around the supposition that ‘If it is one’ (*ei hen estin*) and its negation, and consequences for the ‘one’ (or ‘unity’) are first deduced (1st Hypothesis), that are at first negative (first series of deduction) and then positive (second series of deduction), and so forth, and organized around the distinction between ‘in itself’ and ‘in relation to others’, to give the following structure:

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1 This approach depends on taking the phrases *pros heauto* and *pros ta alla*, i.e., ‘in relation to itself’ and ‘in relation to the others’ to have their a non-technical, straightforward meaning, and differs in this from (Meinwald 1991, chap 3). For a recent defence of the ‘straightforward reading’, see (Rickless 2007, 102-106).

2 This reading is hardly new, it was suggested by the historian of Greek mathematics, Sir Thomas Heath in (Heath 1921, vol. 1, 273-283), and developed since in particular by Kirk & Raven in the first edition of *The Presocratic Philosophers* (Kirk & Raven 1957, 286-297), and G. E. L. Owen in ‘Zeno and the Mathematicians’ (Owen 1986).

3 I am discounting here, and in the schema below, a third series of deduction under the 1st hypothesis, at 155c-157b, of the form ‘A & ~A’, which is reminiscent of Parmenides’ ‘third way’ in his *Poem*. I am also discounting Neoplatonist readings, which focus on the first deduction of the first hypothesis as an explanation of negative theology, revealing an unknowable, ineffable God, beyond reality and simply destroy the dialectical structure of the second part of *Parmenides*. For a critique see (Dodds 1928) and (Allen 1983, 189-195).

4 This reconstruction is partly based on (Gill & Ryan 1996, 57-58) and (Brisson 1999, 46).
The dialectic structure goes deeper than the mere game between the two characters (the old Parmenides and a young Aristotle). Since suppositions such as ‘If it is one’ are not self-contradictory, one needs, in order to derive contradictions within each series of deductions, to consider the bearing of the suppositions on a set of qualifications, e.g., part/whole, limited/unlimited, motion/rest, equal/unequal, etc. As Reginald Allen noticed, the list of these qualifications “reflects the Eleatic tradition”, as the list is fully represented in Parmenides’ *Poem*, and “may indeed conform to the principal divisions of Zeno’s book”. One could add here the qualifications in Gorgias’s *On Not-Being*. A possible topic for further investigation in this connection would be to try and argue, in the footsteps of J. L. Ackrill, that this dialectical context is the source of Aristotle’s ‘categories’.

Of course, I cannot even begin to argue properly for any of the above claims about Zeno and Plato’s *Parmenides*. These two examples were only introduced in order to illustrate what I called earlier the ‘method of dialectical games’, so that one can see how the Greeks structured their arguments in a way that some of Plato’s other dialogues, especially the early so-called ‘Socratic’ ones, do not explicitly show, given that only one thesis (or a series of theses not explicitly contradicting each other) is tested.

In order to emphasize the importance Aristotle assigned to this method, it is worth pointing out that he also believed that the ability ‘to go through the difficulties on either side’ is a necessary condition for philosophy, i.e., a task that philosophers cannot do without first fulfilling it. This is expressed at *Topics*, Θ, 14, 163β9-16:

[…] when it comes to knowledge and the wisdom that comes from philosophy, being able to discern – or having already discerned – the consequences of either assumption is no small instrument; for it remains to choose one or the other of these rightly. In order to do that, one must be naturally gifted with respect to truth: to be properly able to choose the true and avoid the false. This is just what the naturally good are

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1 On self-contradictions within dialectic, see the remarkable study (Castagnoli 2010).
3 (Ackrill 1963).
able to do, for it is by loving and hating in the right way whatever is presented to them that they judge well what is best.

There is an exact correspondence with Aristotle’s own practice, as he begins his treatises in this way.¹ This is the case, e.g., for his investigations on time, in Physics, Δ, 10, where the arguments adduced are clearly dialectical, or for his investigation of *akrasia*, in Nicomachean Ethics, where he write at H, 1145b2-7:

*We must, as in all other cases, set out the phenomena before us and, after first [going through all the] difficulties, go on to prove, if possible, the truth of all the reputable opinions about these affections or, failing this, of the greater number and the most authoritative; for if we both resolve the difficulties and leave the reputable opinions undisturbed, we shall have proved the case sufficiently.* (Translation slightly modified)

There are further *aporia* of the form ‘Is it A or ¬A?’ Aristotle’s treatises, e.g., in De Caelo, I, 10, 297a4f., where the question is ‘Is the world eternal or not?’², and, most significantly, in Metaphysics Book B contains a discussion of 14 such *aporia*, about which Aristotle does not take sides.²

4. Concluding Remarks

The main claim of this paper is, again, that the context within which one should try and understand the emergence of syllogistic is that of the practice of dialectical games. There are already indications within Prior Analytics that Aristotle conceived of his syllogistic within that context, e.g., his distinguishing at the very outset (A 1, 24a22f.) between ‘demonstrative’ and ‘dialectical’ premises, or this advice to the players at B 19, 66a25-31:

*In order to avoid being defeated with a deduction, one should take care, when someone is asking for the premises without the conclusion, not to allow the same thing twice in the premises, since we know that a deduction does not come about without a middle and that the middle is what is said several times. And the way one must watch out for the middle with relation to each type of conclusion is evident from a knowledge of what sort of conclusion is proved in each figure.*

There are also further indications in Posterior Analytics, e.g., at B 3, 90b35-38, of first ‘going through the difficulties on either side’, in order, so to speak, to provide the raw material:

*Now, that everything we seek is a search for a middle term is clear; let us now say how one proves what a thing is, and what is the fashion of the reduction, and what definition is and of what, first going through the puzzles about them.*

To conclude, I would like to offer two reasons, having to do with points (3) and (6) above, why switching to game semantics offers a better framework for the study of the history of logic. First, concerning point (3) and Łukasiewicz’s claim that Aristotle had no clear idea of the quantifiers. This is certainly true if he meant Fregean quantifiers, but

¹ For this point, see, e.g., (Le Blond 1939, 44), (Brunschwig 1967, xvi-xvii), or (Smith 1999, xvii-xix).
² See (Crubellier & Laks 2009).
false otherwise. There is a very clear meaning explanation of universal affirmative propositions at Prior Analytics, A 2, 24\textsuperscript{a}28-29:

*We use the expression ‘predicated of every’ when none of the subject can be taken of which the other term cannot be said.*

This can be interpreted as a ‘no counterexample’ interpretation. In a forthcoming paper with Helge Rückert,\textsuperscript{1} we show how this meaning explanation is derived from a rule for dialectical games at Topics, Θ, 2, 157\textsuperscript{b}34-157\textsuperscript{b}2:

*When it happens that, after you have induced from many cases, someone does not grant the universal, then it is your right to ask him for an objection. However, when you have not stated that it does hold of some cases, you have no right to ask ‘of which cases does it not hold?’ For you must previously carry out an induction to ask for an objection in this way.*\textsuperscript{2}

The rule states that one can only introduce an universal affirmative proposition only after having had the other player concede instances, and that this player must in turn either grant it or provide a putative counterexample. We also explain how this rule is related to the rule in modern dialogical logic for universal quantification, presented informally above. One should note further that this rule is used twice in Plato’s Lesser Hippias (366c-369b and 373-376), a dialogue explicitly mentioned by Aristotle in Metaphysics, Δ, 29, 1025\textsuperscript{h}6-13 – he even attributed to Socrates the invention of inductive arguments in Metaphysics, M, 4, 1078\textsuperscript{b}28.

As a consequence of this approach, one is in a position to explain the doctrine of the existential import of the quantifiers as stated, e.g., in Topics, Γ, 6, 119\textsuperscript{b}34. It is not sufficient that one merely notes, as is usually the case, its presence in Aristotle’s syllogistic: one should be able to explain why it is there. One possible explanation would thus be that Aristotle did not think of the case where the domain of quantification would be empty precisely because the dialectical rule always requires that one establishes a universal proposition on the basis of a number of instances. So universal propositions never come into play without instances being established first. These points illustrate the superiority of an approach via game semantics.

Another illustration could come, if the point were to be argued fully, from point (6) above, concerning Aristotle’s remarkable claim that one can always recover a direct syllogism from *per impossibile* one and *vice-versa*. Indeed, if *per impossibile* syllogisms are those occurring within dialectical games, then this is a crucial claim: it would allow us, first, to understand how Aristotle could *extract* from the practice of dialectical games the very rules of his syllogistic, as shown with the above example:

*Suppose that ‘Not all b are d’, then since ‘All b are c’, it follows that ‘Not all c are d’, but ‘All c are d’, therefore ‘All b are d’.*

\textsuperscript{1} (Marion & Rückert unpublished).

\textsuperscript{2} The rule is also stated at Topics, Θ, 8, 160\textsuperscript{b}1-6.
From this *per impossibile* syllogism one can extract the two universal premises ‘All $b$ are $c$’ and ‘All $c$ are $d$’, from which the conclusion ‘All $b$ are $d$’ follows. Of course, in the game, these two premises would have to be introduced in accordance with the above rule.

Secondly, one would also be in a position to think of Aristotle’s *topoi* as strategies for getting one’s adversary in these games to concede these key premises, that are indeed common to both syllogisms; as Jacques Brunschwig so aptly put it, the purpose of a *topos* is “to frame premises from a given conclusion”.¹ Here, the claim ‘Not all $b$ are $d$’ would be that with which the game starts, with $P$ asserting it. Thus $O$’s task would be to get $P$ to concede the contradictory ‘All $b$ are $d$’, by deriving some obviously false claim such ‘Not all $c$ are $d$’ (in the case of Zeno’s arguments, the derived claim was that there is no motion).

These two last points are speculative, if they were to be convincingly argued, one would then reach a better understanding the true nature of the relation between *Topics* and *Prior Analytics*, improving thereby our understanding of the origin of logic. As an offshoot, one would also gain a better understanding of the reasons why dialectic, under one guise or another, survived alongside syllogistic until the 18th century. And these are reasons too for studying this neglected aspect of Greek, Medieval and Arabic logic.

Bibliography


¹ (Brunschwig 1967, xxxix). Similar suggestions have been made, prior to Brunschwig in (Kapp 1975, 39) and, more recently, (Slomkowski 1997, 3).


