



IBN SINA, FREGE AND THE GRAMMAR OF MEANINGS²¹

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Abstract

It was often assumed in Aristotelian logic that sentence meanings are built from word meanings rather as sentences are built from words. Two logicians who described the grammar of meanings in detail were Frege and Ibn Sina, Frege in *Begriffsschrift*, Ibn Sina in many remarks throughout his logical writings. *Begriffsschrift* is widely available, but the task of extracting Ibn Sina's account from his Arabic is still in progress. The paper, summarising a fuller report on the web, compares the views of Ibn Sina and Frege on the grammar of meanings, and discusses issues raised by places where the two disagree.

ملخص.

لقد وقع الاقرار غالبا في صلب المنطق الأرسطي بأن دلالات القضايا تتكوّن من دلالات الألفاظ بدلا من أنّ القضايا تتكوّن من الألفاظ. إثبات من المناطق اللذان قاما بوصف تركيبية الدلالات بصورة مفصلة هما فريجه و ابن سينا، الأوّل في كتابه : *اللغة الرمزية*، و الثاني من خلال عدّة إشارات في مؤلفاته المنطقية. الكتاب الأوّل منتشر لدى الكثيرين، و لكن مهمّة استخراج المقاربة السينوية من مؤلفاته العربية لاتزال في طور الإنجاز. سنحاول في هذه المقالة ، التي تلخص بحثا أطولا منشورا على الواب، مقارنة آراء ابن سينا و فريجه بخصوص نحو الدلالات و مناقشة المسائل التي يختلفان فيها.

Résumé.

Il a été souvent assumé dans la Logique aristotélicienne que les significations des propositions sont construites à partir des significations des mots plutôt que les propositions sont construites à partir des mots. Deux des logiciens qui ont décrit de façon

²¹ This paper is a summary of the conclusions of a longer paper, [1] which is on my website at <http://wilfridhodges.co.uk/arabic25.pdf>. Because of the limitations of space and the doc format, this summary is drastically simplified. I left out the detailed acknowledgements, the supporting references (there are many), the diagrams and some of the accents. These omissions made the paper easier to write, and I hope they make it easier to read too. I thank Hamdi Mlika for his kind invitation to submit this paper to Al-Mukhatabat.

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détaillée la grammaire des significations étaient Frege et Ibn Sina, Frege dans *Begriffsschrift*, et Ibn Sina dans plusieurs notes à travers ses écrits logiques. *Begriffsschrift* est largement accessible, mais la tâche d'extraire la théorie d'Ibn Sina de ses écrits en arabe est en cours. L'article qui est un résumé d'une étude plus développée accessible sur le web, comapre les idées de Frege et Ibn Sina sur la grammaire des significations, et discute les enjeux de leur désaccord.

Given the present state of our understanding of Ibn Sina's logic, any account has to be regarded as provisional. We make progress through (i) narrow studies of particular texts, (ii) narrow studies of particular aspects or ideas, (iii) comparison with Ibn Sina's likely sources and (iv) connecting up the information given by (i), (ii) and (iii). The present line of work is an example of (ii).

Throughout the paper, a sentence is a syntactic object, and a proposition is the meaning of a sentence. Some other notation and terminology will be introduced as needed. Terminology connected with dependency grammar is explained in section 3.

1. Ibn Sina and Frege as logicians

Ibn Sina in the 11th century and Gottlob Frege in the 19th century both wrote a great deal about logic. Frege's single most substantial logical work is his two-volume *Grundgesetze der Arithmetik* (1893 and 1903), which runs to nearly 520 pages; but in this paper we will be more concerned with his shorter work *Begriffsschrift* (). Ibn Sina's longest surviving logical work, though probably not the longest work that he wrote in logic, is his commentary on Aristotle's *Prior Analytics*, entitled *Syllogism (Qiyas)*; it takes up 580 pages in the Cairo edition. For this paper we will use material from *Qiyas* seven of his other logical works, chiefly his commentary *Ibara Aristotle's De Interpretatione*. All these works were written in Arabic, though Ibn Sina did also write at least one Persian work touching on logic. Both Ibn Sina and Frege learned their logic in the Aristotelian tradition, and both were rebellious. Frege's rebellion was much more thorough than Ibn Sina's. He had the advantages of being a better mathematician than Ibn Sina, and of living in an age when mathematicians had done much of the groundwork towards a new logic. Also Frege was only one



of several logical innovators working largely in parallel, whereas Ibn Sina certainly suffered from having no sparring partners at his own level.

Frege's reputation rests almost entirely on his work in logic and the foundations of mathematics. By contrast Ibn Sina would have been world famous even if he had never written a word of logic; he has been read mostly for his metaphysics and his medicine. But one shouldn't assume that Ibn Sina's logical writings contain any more metaphysics than Frege's. The opposite may be true. Ibn Sina accepted that if logic is to be built on foundations, these foundations will have to lie in a study of the elements of thinking and communicating, an area that is common ground between logic and metaphysics. But he thought that the practice of logic needs very little from this area, and certainly less than philosophically minded logicians usually reckon. On the basis of his discussion of other issues, one can guess that he would have found Frege's distinction between object and function too ontological for comfort. He would very likely also have been unhappy with Frege's lack of interest in how we can tell that functions are not objects, particularly when Frege was so anxious to determine how we know that one plus one is two.

On paper, Ibn Sina and Frege agreed that logic should be kept distinct from both psychology and linguistics. But here again there are differences. Frege tended to speak of natural language as an enemy of logic; in fact he invented a new language, *Begriffsschrift*, because he 'found the inadequacy of language to be an obstacle' to the precise thinking that he aimed at in logic. Ibn Sina said many times that the peculiarities of this or that language are a matter for the linguists and not the logicians. But he also insisted that we can think only with words, and that we establish the rules of correct thinking by a close critical analysis of how we actually do think. Logic rests on some basic intuitions about what words mean and what follows from what. If we move to an artificial language (and he was very conscious that some Aristotelian logicians had tendencies in that direction), we lose the source of our intuitions. For this reason he was even suspicious of moving from a sentence p sentence of the form ' p true in situation s '. His comment on this move was 'You had the thing in your two hands and you just throw it away'.



2. Compositionality

Ibn Sina and Frege shared a certain view of language. According to this view, when I talk to you, I convey to you the meanings that are in my mind. Meanings are compound and built up of parts. I encode the meanings in my mind by using words to encode the smallest units of meaning, and combining the words in a way which encodes the way in which the part meanings were combined. You understand me by decoding my sentences back into meanings. Neither Ibn Sina nor Frege invented this view. It was articulated in the Arabic world by Ibn Sina's 10th century predecessor Al-Farabi, and similar views were expressed in the West by Abelard, Ockham and Leibniz. Ockham even spoke of a mental language, anticipating the name of Frege's book.

We can call this view of language 'Aristotelian compositionality'. It should never be confused with PTW compositionality, the notion of compositionality associated with Barbara Partee and her colleagues (Partee, Ter Meulen, Wall) in linguistics and Donald Davidson in philosophy. Aristotelian compositionality involves the notion that meanings have parts, as in Frege's letter to Jourdain: "The possibility of our understanding propositions which we have never heard before rests evidently on this, that we construct the sense of a proposition out of parts that correspond to the words'. By contrast PTW compositionality, which traces back to Tarski and his truth definition, rests on the notion of an autonomous recursive syntax, and it makes no use of 'parts of meanings'.

Aristotelian compositionality allows us to transfer grammatical notions from expressions to meanings, so that we can speak of a grammar of meanings. A number of Aristotelian logicians pursued this idea. Al-Farabi was deeply interested in lexicographic questions related to logic, and Ockham freely borrowed grammatical categories in order to classify mental words. But Ibn Sina and Frege are unusual in the degree to which they used syntax (not just

grammar) in the realm of meanings. They both studied what Frege called the *Satzzusammenhang* — the way that the parts of a sentence interlock to produce the form of the whole. This increases the chances that comparing the grammars of meaning put forward by these two logicians will turn out to be a useful exercise on both sides.



Frege described the grammar of meanings by developing an artificial language, *Begriffsschrift*, in which each sentence meaning is represented by a 'formula'. He gives us a precise description of this language, and many pages of examples. Ibn Sina never developed a special language to represent meanings, and we saw in the previous section why he would not have approved of any project to do this. But we do have other ways of learning how he thought of the structures of compound meanings. In the first place there are his own general descriptions of the kind of compound involved. He liked to compare constructing a compound meaning with building a house: the foundations are put down, and then other pieces are attached one by one to what has already been built. In the second place he has a large vocabulary for talking about 'attaching' or 'adding' or 'adjoining' or 'linking' meanings to each other. He particularly likes to talk of adding meanings as 'conditions' on other meanings. He also describes meanings as being 'part of the subject' or 'part of the predicate'; this belongs to the same circle of ideas, because as he explains, 'If we adjoin any condition to the predicate, it is a part of the whole predicate'. I call these words 'attachment words'. None of them are unique to Ibn Sina, but their frequent use in combination is very characteristic of him and probably identifies him uniquely among Arabic logicians. Modern glossaries of Arabic or Avicennan logic generally ignore these words or mention them only in other uses.

We have to be careful when we draw inferences about meanings from Ibn Sina's use of attachment words. One major problem is that Ibn Sina uses these same words to talk about attaching expressions to each other in order to form sentences. In spite of compositionality, this is not the same notion, because the structures of expression and meaning generally don't match exactly. Ibn Sina was fluent in both Persian and Arabic, which are very different languages. He knew that often the same meaning is expressed by differently structured sentences in the two languages. For example he points out that the order of words varies from one language to another, and so the corresponding meanings have no intrinsic order. Different languages put possessive constructions ('A of B') in different orders. Some languages put predicates before subjects. So Ibn Sina is very conscious that there can be differences between the structure of a compound expression and the structure of its meaning, and often he says explicitly which he is talking about. But not



always; sometimes the distinction is obvious from the context and sometimes it's irrelevant.

Today we have quotation marks to make clear when we are talking about expressions. It's helpful to have a kind of semantic quotation mark to show that we are talking about meanings. Following Jackendoff I write [HORSE] for the meaning of the word 'horse', and [EVERY HORSE] for the meaning of the phrase 'every horse'. Ibn Sina had a device that can sometimes serve the same purpose. He names the sense of a word *W* by writing '*W*-ness', or in Arabic '*W-iyya*'. This produces some curious results, like 'itness' for [IT] and 'not-a-horse-ness' for [NOT A HORSE]. The usual universal quantifier is *kull*, so he names the meaning [EVERY] by writing *kulliyya*. But some disastrous choices of notation, not all Ibn Sina's fault, immediately cause trouble. First, nouns in *-iyya* also used to form nominalisations from verbs and adjectives; so *kulliyya* means 'being universally quantified'. Second, in Aristotelian logic we also have the notion of a universal, i.e. the meaning of a common noun, and the Arabic for 'universal' is *kulliy*. So *kulliyya* stands for [UNIVERSAL] and for 'being a universal'.

Here is a remark of Ibn Sina about the meanings [EVERY ANIMAL], [SOME ANIMAL] and [THE ANIMAL]. It illustrates the pitfalls.

We say: The idea [ANIMAL] is a meaning in its own right, regardless of whether [an instance of] it exists in the world or it is conceptualised in the mind. Also it is not in itself either universally quantified or existentially quantified. If it was inherently universally quantified, in the sense that the meaning [ANIMAL], just as it is, was universally quantified, then there couldn't be an individual animal. Rather, each animal would be an every animal. And if [ANIMAL] were singular, just because it

was [ANIMAL], then it wouldn't be possible for there to be more than one individual [animal]. Because of [ANIMAL] being singular, there would have to be just one animal, and it would be impossible for any other individual to be an animal. ... If together with [[ANIMAL]] one conceptualised [EVERY] or [SOME] or something else, then what would be conceptualised



would be a meaning added to [ANIMAL], something that happens to [ANIMAL].

This is from page 65 of *Madkhal*, Ibn Sina's commentary on Porphyry's *Eisagoge*. The passage is often quoted and widely said to be 'paradoxical'. I hope the reader will agree that there is nothing even faintly paradoxical about it. It's a very straightforward statement about certain phrase meanings; Ibn Sina makes similar remarks about these phrase meanings in other places. Apparently many readers ran into paradox by missing the right interpretation of *kulliyya*.

Actually there is one feature of the paragraph that might puzzle a modern reader, namely that Ibn Sina seems to jump between existentially quantified (as in 'some animal') and singular (as in 'the animal' or 'this animal'). The reason becomes clear when we look at his treatment of existential quantification elsewhere. He regards 'Some B is an A' as true when a certain B, say *b*, is an A. The sentence 'The B is an A' is true under the same condition; the difference is that in the first case it's not determined which individual *b*, whereas it is determined in the second case. In modern jargon we can express this by saying that Ibn Sina's semantics for existential quantification is closer to a Skolem function semantics than a Tarski-style one. A number of modern semantically-minded linguists have developed similar views.

This example also illustrates Ibn Sina's interest in methods in semantics. How can you prove (say, to your younger sister who is not a logician) that 'animal' and 'every animal' have different meanings? It's a question that he keeps coming back to from different angles. Probably he was the first semanticist to ask the question in this form.

3. Dependency grammars

It seems to be an old and natural idea that sentences are built up by taking a word and attaching words to it, then attaching further words to those, and so on until the sentence is complete. If we analyse a sentence by identifying which words were attached to which as it was constructed, the result can be written as a tree branching downwards: the first word is at the top, and to attach a new word we write it below the word that it's attached to, with a line



connecting them. A word is said to be 'dependent on' a word that it's attached to, though this is a purely formal notion and different grammarians have used it to represent different relations.

A diagram of this kind is known as a 'stemma' (plural 'stemma'), after Lucien Tesnière who in the mid 20th century advocated this style of sentence analysis. A grammar that analyzes into stemmata, or anything equivalent to them, is called a 'dependency grammar'. Dependency grammars have been around for centuries. Ivor Mel'čuk, a modern dependency grammarian, claims that one can trace them back to Antiquity. The 13th and 14th century Modist grammarians had a dependency grammar framework. Jonathan Owens has claimed that classical Arabic sentence analysis was implicitly done in a dependency grammar, though this has been disputed. Analysis by stemmata was widely used in schools in Germany in the 19th century, and almost certainly Frege would have been taught it.

Stemmata are labelled graphs: the words are labels on the nodes, and the connecting lines are the graph edges. So we can apply some graph-theoretic terminology. Thinking of the edges as 'attachments' gives them a direction, so it's natural to take the graph as directed, with the edges as arrows pointing upwards. The top node is the 'root' (remember that the tree branches downwards). A 'branch' of the stemma is a line of nodes and edges that travels from the root to a node with no dependents, going downwards along the edges. The 'in-degree' of a node n is the number of arrows with their heads at the node; so it's equal to the number of nodes dependent on n . A 'principal subgraph' of a stemma consists of a node and all the nodes and edges that can be reached from it by going downwards along edges. When a stemma analyses a phrase, then a 'principal subphrase' consists of all the words in some

principal subgraph; this collection of words normally does form a phrase. The 'head' of a principal subphrase is the word at its top node. This term 'head' comes from linguistics rather than graph theory; the linguists also call the word at the root of the stemma the 'head' of the phrase analysed by the stemma.

The formulas of Frege's *Begriffsschrift* stemmata, slightly tilted so that they branch to the east or the south-east rather than to the south. So *Begriffsschrift* is a sort of dependency grammar, where each formula is its own grammatical analysis. At the nodes Frege puts the counterparts of 'words' in his language.



These are negation, 'if ... then', universal quantifiers and atomic formulas.

Each node of a *Begriffsschrift* has an in-degree that is determined by the word that labels the node. Negation and quantifier both have in-degree 1, 'if ... then' has in-degree 2 and atomic formulas have in-degree 0. This is not just an accidental result of Frege's choice of 'words'. In the interpretation of *Begriffsschrift* Frege gave from the early 1890s onwards, each 'word' has a Bedeutung that is either a function or an object. The Bedeutung of a 'word' at a node n a function whose arguments are the Bedeutungen of the 'words' at the nodes dependent on n . Since each function has a fixed number of argument places, each node has a fixed in-degree.

This analysis shows us how we could extend a *Begriffsschrift* downwards by analysing the atomic formulas: each such formula is got by applying a relation to arguments, and the arguments will be at nodes dependent on the node with the relation itself. So the atomic formula becomes a principal subphrase whose head is the relation; the number of arguments of the relation is the in-degree of its node.

It will turn out that Ibn Sina implicitly gives his meaning structures a dependency grammar. It's similar to the grammar of *Begriffsschrift* some ways, but different in others. One difference is that for Ibn Sina the in-degrees are not at all fixed. In fact his methods of formal logic involve changing the in-degrees. But to explain this and the other differences, we will need to spend some time analysing various things that Ibn Sina says about meanings.

Today linguists rarely use dependency grammars. Instead they tend to use versions of phrase markers, which are also trees branching downwards but with a different interpretation. When you analyse a phrase using a phrase marker, the words of the phrase are put at the bottom tips of the branches — these tips are the 'terminal' nodes. Each non-terminal node n a subphrase, namely the phrase consisting of all the words on the terminal nodes that can be reached by going down from n ; the label on n be a symbol describing the grammatical category of this subphrase.

Phrase markers allow two kinds of freedom that we don't have with stemmata. First, a subphrase doesn't have to have a head word. In general there doesn't seem to be any good linguistic reason why every subphrase should have a head word; so it's an interesting and perhaps important fact that for Ibn Sina and Frege phrase meanings always do have heads.



And second, even when a subphrase does have a head word, the phrase marker has distinct nodes for the head word and for the subphrase. So we can distinguish between linking a word to the head of the phrase and linking the word to the subphrase as a whole. For example a grammarian might want to claim that a modal word like 'Possibly' is attached to a whole sentence rather than to a single word in the sentence; we can represent that fact in a phrase marker but not in a stemma. Again it's a significant fact about *Begriffsschrift* Frege arranges things so that he never needs to use this distinction. The position with Ibn Sina is not so clear — there are places where Ibn Sina does seem to want to distinguish between linking to a phrase and linking to its head, and at these points the dependency framework will break down.

To avoid one possible misunderstanding, I should add that some people use dependency grammars to explain not just the grammatical relationships between the words in a sentence, but also the order of the words. We don't do this. When we draw a stemma on the page, the left-right order of the nodes dependent on a particular node has no significance, and it doesn't alter the stemma if we change that order. (In this we agree with Mel'čuk.) For Ibn Sina the order of the nodes dependent on a given node has no significance. For example subject meaning and predicate meaning are different, but they aren't distinguished by order, since different languages disagree about their order. They are distinguished by having different kinds of dependency. Frege in his *Begriffsschrift* left-right order to distinguish 'If p then q ' from 'If q then p ', but his stemmata don't determine for example whether a negation should be written before or after what it negates.

4. Silent meanings

One of Ibn Sina's most characteristic doctrines was that when we speak, we normally leave unspoken some parts of what we mean. Let me call this the doctrine of 'silent meanings'.

Frege would probably have been in sympathy with this doctrine, because one of his chief bugbears was that mathematicians routinely leave out essential parts of their proofs. But Frege and Ibn Sina reacted in diametrically opposite ways. For Frege it was essential to bring the hidden meanings out into the open, even if we have to invent a new language for the purpose.



By contrast, Ibn Sina reckoned that the task of logic is to understand and control reasoning in the sciences and philosophy, not to replace that reasoning by something new. If we abandon the accepted styles of writing arguments, how can we hope to understand them? This attitude leads Ibn Sina into copying even the most grotesque bad habits of his sources. For example he is particularly careless about leaving out the arguments of comparisons, as in 'Equals are greater' (for the false statement that if X is equal to Y then X is greater than Y). Often he leaves out the objects of verbs if he thinks the reader can guess them from the context.

Every time that we say something but leave part of our meaning unspoken, we break the rules of compositionality. For Frege this is simply intolerable. For Ibn Sina it is tolerable and something we do all the time, but a speaker or writer who wants to be understood must take care not to leave out anything that the hearer can't infer. Of course this means that Ibn Sina can leave out more when he reckons he is writing for more intelligent people. There are good grounds for suspecting he sometimes did this deliberately in order to make sure that only the intellectual elite would read him.

What kinds of meaning does Ibn Sina think we leave silent? I give some examples below, but it's by no means a complete list. Section 8 below will add some more examples. Almost certainly Ibn Sina thought that to give a complete list would be an open-ended task.

(1) *Assumptions of the form 'Suppose ...'*. Here Ibn Sina closely tracks Frege's treatment of the same topic in some of his later works. For both Ibn Sina and

Frege, an assumption introduced by 'Suppose' is really an assumption introduced by 'If', except that by saying 'Suppose' we allow ourselves the liberty of not repeating the assumption every time we use it. For Frege this is a dangerous habit, though it's understandable because of the 'monstrous length' that arguments would have if the assumptions were repeated every time. Ibn Sina's view by contrast is that the assumption is really there in the meaning, and at least between competent scientists it's communicated silently. These assumptions made but not repeated seem to me the most likely explanation of the places where Ibn Sina appears to say (for example) that an Arabic sentence whose normal translation would be 'No animal moves' means



that for every animal there is a time at which it's not moving. In spite of the appearances, Ibn Sina can't really mean this. To distort Arabic in this way would run clean counter to his constant requirements that we should observe normal usage. More likely he has in mind discussions between scientists, for example about the nature of sleep. Someone says 'Let's consider animals sleeping', and this assumption remains in force for the rest of the discussion unless someone cancels it. If Ibn Sina had explained this properly, he could have saved us a lot of scratched heads, and himself some embarrassing misrepresentations. He has only himself to blame.

Since Ibn Sina thinks that his own predecessors in logic made use of these unspoken assumptions, he has to allow that a syllogistic mood like Barbara can be understood as covering arguments where an extra assumption is tacked onto some of the sentences. This is one of the cases where he does think it's worth his while to bring the silent assumption into public view. In Book vii of *Qiyas* runs through a number of syllogistic forms and checks to what extent they still work with extra assumptions added in. (This book in *Qiyas* very tedious and I think few people will bother to read it.)

(2) *The second quantifier.* Early in *Qiyas*, Ibn Sina gives a number of examples of sentences, nominally of the form 'Every B is an A', where in fact we would automatically understand them as having a more complicated form. For example

Every white thing has a colour with wide radiation.

(Today we would say 'wide spectrum'.) In this example we automatically assume the speaker means that the white thing has the colour with wide radiation for as long as it stays white; if we paint it blue we make the description false. In case you think this is too obvious to be worth mentioning, compare with Ibn Sina's next example:

Everyone who travels from Rayy (a suburb of Tehran) to Baghdad passes through Kermanshah.

Nobody will regard this as saying that the traveller passes through



Kermanshah for as long as he or she is travelling. A more natural reading is that the traveller passes through Kermanshah at some time during the travel. And in case you think that the time of the predicate description is automatically assumed to be within the time of the subject description, try another example that he gives:

Everything that breathes in breathes out.

And so on. The root cause of all these examples is that many descriptions are 'separable accidents', to use the Aristotelian jargon; a thing can fit the description at time t fail to fit it at time $t+1$. Using descriptions of this type, we always have a time variable, at least implicitly. So some kind of temporal quantification is needed, though again it tends to be implicit.

This analysis in terms of two quantifiers is Ibn Sina's own. He knows that there is a classical account of the meaning of 'Every', and it takes care of the subject quantifier in

Every horse spends time awake.

But he insists that the sentence contains more quantificational structure that still needs analysis; there is a time quantifier and it has to be related somehow to the subject quantifier. Today we would reach for a formalisation that puts the two quantifiers in a certain order, to show which has wider scope. Ibn Sina doesn't have such a formalisation, but his idea of how and where the

various quantifiers are attached to the rest of the meaning should serve a similar purpose. If it doesn't, that's a flaw in his picture of the meaning structure.

(3) *Inhibitory conditions.* In modal logic we qualify propositions with modalities: 'necessarily', 'possibly', 'contingently', 'it's impossible that ...'. A proposition with no such condition is said to be 'absolute'. But now suppose that I utter a sentence, and as I utter it I make the deliberate decision that it should not be understood as having any modal condition. This is my decision about the meaning of my sentence, and for Ibn Sina the speaker's intention is always



decisive for the meaning. So somewhere in the meaning of the sentence there is an item specifying that it's absolute, in other words a condition telling the hearer not to add any modal condition.

This way of thinking emerges in the many places where Ibn Sina treats 'absolute' as another modality alongside 'necessary' and 'contingent'. In section 6 below we will see where he puts modalities in his meaning structures; he puts 'absolute' in the same places as the other two modalities.

Another curious example of inhibitory conditions appears in Ibn Sina's treatment of counterfactual conditionals. (I adjust his example a bit.) We can ask: If five was two plus two, would it be an odd number? The example is a little similar to a modern one about whether, if Caesar had been alive now, he would have used arrows. One might say that by assuming five is two plus two we deprive ourselves of the right to use the information that five is odd; in this sense, counterfactual reasoning is non-monotonic. Ibn Sina is sympathetic to that view. But he also realises that it would wreak havoc with scientific uses of *reductio ad absurdum*, where we make a false assumption precisely in order to deduce a contradiction from it together with known facts that are incompatible with it. So he suggests that when making scientific theses, we may intend a strong form of assertion that forbids us to overrule the consequences of these theses. He says we 'add to the clause in the meaning a condition that prevents the use of conditions entailing things that are not allowed to follow from the clause'. I am not sure that either Ibn Sina or I have got all the details of this right, but the general idea peeps through. We can add to the meaning a condition that prevents certain other conditions from being added. A general point that emerges from (3) is that Ibn Sina has broader and narrower notions of what counts as a meaning. In the narrower sense a 'meaning' is a descriptive criterion that divides all possible entities into those that satisfy it and those that don't; the meaning [HUMAN] is of this kind. But in a broader sense anything that I intend when I speak can count as a 'meaning'. The grammar of meanings has to cover meanings of this broader kind.

5. Simple propositions and the copular meaning

We can now begin to describe the structures that Ibn Sina gives to the



meanings of particular sentences. We start with simple sentences, i.e. ones that contain no subclauses. The paradigm examples are sentences like 'Zayd walks' and 'Zayd doesn't walk', where there are a single subject word 'Zayd' and a single predicate word 'walk(s)'.

Ibn Sina presents his account of these sentences in his book *Ibara*. This book is a commentary on Aristotle's *De Interpretatione*, and the context that Ibn Sina chooses is where Aristotle mentions the question of the unity of the proposition. This is a question that goes back to Plato, who asked what is the difference between the sentence 'Theaetetus sits' and the list of two words 'Theaetetus sits'. Ibn Sina discusses the question, but I doubt if he has any interest in it, at least wearing his logician's hat. It's typical of the kind of ontological question that he prefers to see removed from logic. He doesn't mention it at all in the parallel passages in his surviving later works in logic, where he follows his own agenda rather than Aristotle's.

Ibn Sina explains that the meanings of sentences of this simple form have three components. Two of these are the meaning of the subject word and the meaning of the predicate word. The third component carries the extra information that is needed to construct a sentence meaning from the first two components. This information has three parts. We need to know (i) which meaning is the subject meaning and (ii) which is the predicate meaning; and (iii) we need to know the quality of the sentence, in other words, whether it's affirmative or negative.

Ibn Sina is quite right that (i) and (ii) are needed. It's no objection to say that the order of the two words in the sentence determines which is the subject and the predicate. The order of the words is a piece of syntax; the word meanings don't have an order. If a language has the property that it puts subject words before predicate words, then this is simply how that language represents the information (i) and (ii).

Ibn Sina takes the view that (i), (ii) and (iii) need an element of the meaning to carry them, but it can be the same element for all three. He has no name for this element (apart from 'third component', an expression that he takes from Aristotle). I will call it the 'copular meaning', because Ibn Sina goes on to explain that in those languages that put a copular verb like 'is' in the middle of these sentences, the copular verb can be regarded as standing for the copular meaning. If we pressed him on why he thinks that the verb stands for this



meaning, I guess he would say first that verbs of this type have a syntactic subject and a syntactic complement, and these are precisely the words whose meanings are the subject and predicate meaning, giving (i) and (ii); and that such languages generally negate the sentence by attaching a particle of negation to the copular verb, which takes care of (iii). But it's not an important question, because many languages including Arabic have simple sentences with no syntactic components except the subject and predicate words.

Having made this decision to put the information onto a single copular meaning, Ibn Sina needs to link the three elements together somehow: which is connected to which? His remarks about this are fairly equivocal; he uses attachment words like *nisba* h are ambiguous about the direction of attachment. But in fact the symmetries of the situation make only one choice reasonable. If the copular meaning was attached to the subject meaning, it should be attached to the predicate meaning too, so that we would have an element attached in two places, contrary to his general picture. Likewise if the copular meaning was attached to the predicate meaning. The only sensible choice is that both subject meaning and predicate meaning are attached to the copular meaning. Since the copular meaning distinguishes which is subject and which is predicate, we should think of the copular meaning as having two labelled sites of attachment; in dependency grammar this would normally be handled by labelling the two edges as subject edge and predicate edge.

Ibn Sina could also have introduced an optional fourth element, negation, that is attached to the copular meaning in negative sentences. Instead he opts for the view that there are two kinds of copular meaning, one affirmative and the other negative.

There are several decisions here that might seem arbitrary. For example if items (i), (ii) and (iii) can be combined in a single element of meaning, why shouldn't that element be the subject meaning or the predicate meaning? In fact Frege took exactly this view with such sentences. For him the copular function goes with the verb, in this case the predicate. Since the *Bedeutung* of the verb is a function and the *Bedeutung* of the subject word is an object that serves as an argument of the function, Frege's conventions imply that the subject meaning [ZAYD] attaches to the verb meaning [WALKS], which has in-degree 1. Frege seems quite emphatic that this is the only way to go.

I think today most semanticists would regard the difference between Ibn Sina's



picture and Frege's as a purely theory-internal one. We can choose whichever theory works more smoothly in the context where we want to use it. For example Richard Montague found it convenient to choose a third option that makes [ZAYD] the function and [WALKS] its argument. In any case there are systematic translations between one theory and another, using devices like the lambda calculus. This position looks to me closer to Ibn Sina's pragmatic view than to Frege's dogmatism.

Ironically, the authors of one modern semantics text present a semantic structure for sentences of this simple kind which they describe as Frege's, when in fact it's Ibn Sina's. More precisely they display a three-node stemma that corresponds to Ibn Sina's three elements. If we apply their account to the sentence 'Zayd walks', we find that the meaning [ZAYD] is Zayd, the meaning [WALKS] is a function that can take Zayd as argument, and the meaning at the root node is the evaluation function that applies [WALKS] to Zayd.

6. Quantifier and modality

Ibn Sina says that quantifier and modality can be attached to simple sentences. He adds that the quantifier is attached to the subject; the modality can be attached at any of several sites. These are remarks about words. They create a slight presumption that Ibn Sina thinks the same is true about the meanings of the words in question.. But one should check that he really does think this.

In fact the checking is easy. The paragraph quoted in section 2 above is only one of many places where he makes it clear that he thinks of quantifier meanings as attached to noun meanings and forming a compound unit of meaning. Also he applies this treatment to 'it', 'this' and 'that' in the same way as to quantifiers.

When he first introduces modalities in his *Ibara*, Ibn Sina says that purely as a matter of the sentence syntax, the modalities can be attached in two different places. But then he adds that in fact this also makes a difference of meaning. Strictly he could be telling us only that Arabic has certain conventions for indicating modal meanings; but in context this is a very implausible reading of his text. The only plausible reading of what he tells us is that the two possible sites of attachment for a modality meaning in a simple proposition are the copular meaning and the quantifier meaning.



Ibn Sina doesn't say so explicitly, but the quantifier in the second case must be the main subject quantifier and not a silent time quantifier. We know this since it will emerge in a moment that the modality added to the quantifier can in fact consist of the time quantifier. He also speaks of having 'truth on the quantifier' and 'absoluteness on the quantifier'; these seem to be the same thing, regarding absoluteness as a kind of modality.

Ibn Sina adds that Aristotle in his modal logic never allowed modalities on the quantifier; 'in fact the prohibition of them is total'. So for explaining Aristotle — and this is nominally the purpose of *Ibara Qiyas* — the primary site for the modality meaning has to be the other one, namely the copular meaning. But when Ibn Sina explains the meaning of a simple sentence with a modality in this position, he makes clear that although the modality is attached to the copular meaning, it modifies the meaning of the predicate. For example if we add possibility to the sentence 'Zayd walks', we get a sentence meaning that Zayd has the property of possibly walking. According to Ibn Sina's further explanations, if we put 'Every human' in place of 'Zayd' in this sentence, we get a sentence meaning that it's true of every human that he has the property of possibly walking.

There is a puzzle here. It seems that the placing of a word meaning within the sentence meaning should tell us something about the role of the word meaning in determining the meaning of the whole sentence. So if the effect of a quantifier 'possibly' is to modify the predicate meaning, we would expect the quantifier meaning to be attached to the predicate meaning, not the copular meaning. Why does Ibn Sina say otherwise?

This is one of a number of places where it's hard to be sure whether to describe Ibn Sina's thoughts as a muddle or an insight. A case could be made for insight, more precisely for insight into deeper facts of linguistic structure. Many modern syntacticians claim to detect a deep level 'functional category' that sits above a clause and controls the interrelations of the contents of the clause. They describe this category as the site where various things are attached, including modality and sentence negation. They often add tense, which tallies with Ibn Sina's observation that a copular verb determines the time of the clause containing it. This functional category does have a more than passing resemblance to Ibn Sina's copular meaning. But you might still prefer to vote for muddle.



Ibn Sina mentions in a few places that we can nest modalities. One example that he gives is the sentence 'Every human necessarily can be a writer'. He says that we can accommodate the modality 'necessarily' by moving the 'can' from the copula to the predicate, and then attaching 'necessarily'. I don't know exactly what he is talking about here, because his example doesn't seem to match his comments on it. But note that he handles the nested modalities by attaching first the one of smaller scope, and then the one of wider scope. This is one of several places where he could be thinking that where there is a possible clash of scopes, we can make one of the competing items have wider scope by attaching it further out, i.e. lower down. In section 9 below we will come back to this.

We return to the modality 'on the quantifier'. Ibn Sina gives several examples to illustrate this. But they don't entirely clarify what he means. To my eye, the main feature that runs through all the examples that he gives is that there is a wide-scope quantifier over times or situations. (He tends to describe possible situations as 'a time in which'. For example we have 'a time when everything is coloured black'. I think putting this in terms of 'times' is a hangover from Aristotelian habits.) Roughly, a proposition with a possibility modality 'on the quantifier' is one saying that there is a possible situation in which XYZ holds; and one with truth 'on the quantifier' says that in such-and-such a situation XYZ holds.

Suppose the modality on the quantifier really is a quantifier over times or situations. Then maybe Ibn Sina is attaching this quantifier below the subject quantifier in order to express that it has wider scope than the subject quantifier. That would match the earlier example about nested modalities. Again we will come back to this.

I agree with Zia Movahed that in practice Ibn Sina's distinction between modalities on the copula and modalities on the quantifier plays the same role as the distinction in the Latin Scholastics and in modern modal logic between modalities *de re* and modalities *de dicto*. But there is some work to be done matching up Ibn Sina's explanations with the western ones.

7. Compound sentences

Ibn Sina considers compound sentences that are put together out of two



clauses. His treatment is built on how he combines subject and predicate; in fact he says that we can think of the two clauses as analogues of subject and predicate. Though he doesn't spell it out, I think we can assume that he supposes we take the two clause meanings and combine them by attaching them to another copular meaning. Curiously the Arabic translation of Aristotle uses the word for 'copula' only for the case of combining two clauses, not for the case of combining subject and predicate.

I can't resist one further remark about Ibn Sina's treatment of compound sentences. He remarks that in the languages that he knows, the two clauses are labelled with introductory words like 'if' and 'then', which serve to show what kind of combination is being made, and which clause plays which role in it. But then he adds that there might be languages that handle the point differently. Wondering what he can mean, I note that not far to the north of him there were several Caucasian languages that have in a sense no syntactically compound sentences. For example in Abkhaz each sentence has only one finite verb, and things that would be expressed in other languages as subordinate or coordinate clauses are wrapped up in nominalisations. For Ibn Sina this wouldn't affect the meaning structure, but it would be an example of how that structure can be represented in a radically different syntax. Did Ibn Sina meet any Abkhaz speakers and discuss syntax with them? If he did, then maybe they also told him about their ergative verbs. Ergativity could be part of Ibn Sina's linguistic evidence for his claim that there is no natural order for subjects and predicates; though Arabic 'verbal sentences' that start with a verb could also have been in his mind. Ibn Sina's remark about reversing possessive constructions probably refers to Turkish.

8. Adjunctions

One of the skills that an Aristotelian logician is supposed to have is to take any sentence and construct its contradictory negation. In Arabic there is a trivial solution to this problem: just take the sentence and add at the beginning *laysa*, 'it's not the case that'. Actually this doesn't always work, because of some tricky interactions between *laysa* quantifiers, as Ibn Sina mentions. Also Ibn Sina finds it more satisfying to push the negation down inside the sentence, using rules of De Morgan type. In modern jargon, he likes sentences to be



negation-normal.

But for Ibn Sina the trickiest problem about forming contradictory negations is to take care of the hidden meanings. For example if someone says that Ethiopians are black (silently meaning their skin colour), you don't contradict this person by saying that Ethiopians are not black (silently meaning all through their bodies). Quite what this observation has to do with any real-life conversation is a mystery to me. But for Ibn Sina it's an excuse to list several kinds of hidden meaning that might be relevant if you want to avoid the kind of cross-purposes that the example illustrates. Ibn Sina regarded this list as so important that he often referred back to it, even in other books, with phrases like 'the conditions that we said you have to take care of in the case of contradictions'. Some of these back references add other items to the list.

If we are talking just about meanings, then it makes no difference whether the conditions in his list are hidden or explicit. Either way, they are there in the meaning of the sentence. So his list is really a list of conditions that could be added to a proposition. I think he assumes implicitly that the conditions are added to a simple proposition, or at least a single clause in a compound proposition, since he never discusses which of the two clauses of a compound proposition they should be added to. The only sites where added phrase meanings can be attached in a simple proposition are the subject, the predicate and the copular meaning. Ibn Sina never suggests attaching any of these 'conditions that you have to take care of' to the copular meaning. So in effect the list is a list of meanings that can be attached to the subject meaning or the predicate meaning, or as Ibn Sina often puts it, 'made a part of the subject (or predicate)'.

Ibn Sina insists that the conditions need to be taken care of, not just in order to form the correct contradictory negation, but in order for there to be a determinate proposition at all.

The fact is that a proposition in its intended meaning is not true or false at all, or conceded or rejected, or even conceptualised, to say nothing of its having an opposite, unless it is determinate in terms of all the attachments to its meaning that we have mentioned.



That might suggest that the conditions in the list all fill empty argument places in the subject or predicate.

Some of the conditions in Ibn Sina's list do just that. For example to the sentence 'Three is a half' we can add 'of six'. Obviously this fills an empty slot, at least in the meaning.

Other examples are not of this kind. Some add a reference to time, place, circumstance or means. Others modify the meaning of the head word of the subject or the predicate; for example we might add to the sentence 'Alcohol is forbidden' a phrase explaining that 'alcohol' here includes substances that are in process of turning alcoholic.

Some of these conditions, for example those that modify the meaning of a word, are completely optional additions: the proposition would have been thoroughly complete without them. Others create a semantic hole in order to fill it. For example I can tell you that Zayd is a father, and nothing needs adding. But I can add the information that Zayd is a father of Amr. Ibn Sina explains that the definition of 'father' uses the relation 'father of', and that this definition can be opened up to make the second argument place available. An alternative explanation, which matches some other remarks of Ibn Sina, is that the second argument place is always available, but there is a default understanding that we quantify it out — 'father of someone' — unless something is said that overrules the default.

The cases where a condition opens up a new argument place, together with the cases where a condition doesn't fill an argument place at all, account for one of the most significant differences between Ibn Sina's meaning grammar and *Begriffsschrift*. Namely, for Frege but not for Ibn Sina, the in-degree of a node in a stemma is fixed; we can't attach new dependents to the node and still have a stemma. Most languages of formal logic have kept this feature of *Begriffsschrift*, but Ibn Sina's examples illustrate how far removed it is from natural language. I don't know whether Frege imposed a deliberate decision to exclude things like Ibn Sina's conditions, and whether he was just lucky that the mathematical language required for arithmetic never needed any such conditions.

Besides lumping together conditions that fill an empty argument place and conditions that don't, Ibn Sina also interlaces conditions that affect the meaning with conditions that affect the reference. Today, thanks to Tarski,



Quine, Carnap, Montague and others, we tend to make a sharp distinction between assigning reference (a matter of pragmatics) and assigning meaning (a matter of vocabulary).

Or should I have said 'since Frege'? After all, Frege's later work distinguishes sharply between reference (*Bedeutung*) and meaning (*Sinn*). I think this is not the issue. The modern separation normally allows us to distinguish between two occurrences of phrase that are applied to different points of reference without changing the meaning of the phrase. We have a completely meaningful sentence 'The sun is up', but it can be used with reference to Paris at midday or with reference to London at midnight, and it will have different truth values in the two cases. On my reading of Frege he would never allow this. For him the thought expressed by the sentence 'The sun is up' is not complete unless it supplies a time and a place, and the thought can only supply these by having meaningful parts that pin down these references. In this respect, Frege is in complete agreement with the displayed quotation from Ibn Sina earlier in this section.

One modern theory of the structure of noun phrases and verb phrases, X-bar theory, gives a picture not much different from Ibn Sina's. According to this theory there is a head word — a noun in noun phrases and a verb in verb phrases — and various other phrases are adjoined around it. But X-bar theory also predicts that the phrases will be added in a certain order, which depends on the kind of phrase that they are. For example, working out from the noun, adjectives and adverbial phrases will be added before any quantifier. Ibn Sina has nothing about this ordering, except that he seems to add quantifiers first. But serious study of this aspect of noun or verb phrases probably doesn't go back before the twentieth century. All of this is irrelevant to *Begriffsschrift*, which adjoins only those phrases that fill argument places, and these can be added independently in any order.

9. Scope

Frege himself lists some of his main differences from the Aristotelian tradition, and generally these are differences between him and Ibn Sina too. For example he insists that relations can have several arguments. Where Ibn Sina and most Aristotelians would take 'Zayd' to be the subject of the sentence



'Zayd travelled from Rayy to Baghdad via Kermanshah'. Frege would insist that there is no need to give a special role to 'Zayd'. The sentence can be read as consisting of a 4-place relation expression '() travelled from () to () via ()' with four arguments filling its slots. Frege's procedure here is very natural in the light of mathematical practice.

The two other main differences between Frege's *Begriffsschrift* and Ibn Sina's grammar of meanings are the absence of optional conditions in Frege's version, and the different placing of the main quantifier. We discussed the optional conditions earlier. Why did Frege move the quantifier up to the top of the formula that it quantifies? Could it just be that after he had got rid of the subject term, he no longer had it available to attach the quantifier to? Without a subject term he would have had to look for the nearest stable position in the formula, and that happened to be the top of the relevant subformula. I think his choice was much more deliberate and motivated than this, and some work of Ibn Sina helps to clarify the situation.

Recall that for Ibn Sina, a typical proposition of the form 'Every B is an A' has at least one implicit time quantifier in it. So in general there are at least two quantifiers. If they are both universal then we may be able to combine them by quantifying over ordered pairs; Ibn Sina has several examples of this. But if the proposition says something of the form 'For every instance of B there is a time t ...', then the quantifiers have to be kept separate. Ibn Sina comments 'It would be appropriate for us to speak warily. When we try to take the time into account, this causes us difficulties'.

Ibn Sina ran into technical questions that he couldn't answer, and he admitted it. Looking to see what the obstacle was, we realise that Ibn Sina had no notion of scope, at least consciously. He had no notion, for example, that an existential quantifier within the scope of a negation behaves in certain ways like a universal quantifier. He had no notion of what it means for an existential quantifier to be within the scope of a universal quantifier. In all of his two or three thousand surviving pages of logic there is no notion that we can translate as 'scope'.

We easily imagine that the notion of scope was always available. In fact the notion of quantifier scope was invented by Frege as part of *Begriffsschrift*. He didn't get it quite right at first; instead of the scope of a quantifier occurrence, he talked of the scope of a variable, and he explained it in language that is



suggestive but strictly meaningless. He reworked his explanation in unpublished work soon after the publication of *Begriffsschrift*, and by the time of *Grundgesetze der Arithmetik* was well able to explain it soundly, though the shift from scope of a variable to scope of a quantifier came only with Hilbert's lectures on logic. The notion of the scope of a negation first appears in the linguistic literature in the mid 20th century. Paper [2] in the bibliography below discusses the history of scope from Ibn Sina to the late 20th century.

In both the logical and the linguistic literature, the scope of an element E of a sentence is a part of the sentence that is in some sense governed by the element; call this part of the sentence the 'domain' of E. What Frege did in *Begriffsschrift* to put his quantifiers in positions where their domain could be read off immediately from the structure of the formula. A quantifier sits at a node of in-degree 1, so there is a single node dependent on it. The domain of the quantifier is the principal subformula with that node at its head. (Another convention puts the quantifier itself into its scope as well; Frege is equivocal about this in *Begriffsschrift*.) The same definition of the domain works for negations, which also sit at nodes of in-degree 1.

In natural language we usually can't pin down the scopes in such a mechanical way. But there has been strong pressure in linguistics to look for structural criteria for scopes; this was the motivation for the notion of 'command' in generative syntax.

How about Ibn Sina, struggling with his pairs of quantifiers? Suppose he had formed a notion of scope and tried to define it. There are not many ways in which we can read nodes in stemmata as defining subphrases. In a sentence one might think of the scope of an element as the clause following the element; definitions along these lines are often given and quite often they work. But these definitions make sense only because sentences are linearly ordered. Ibn Sina will naturally want to define scope in terms of the meaning structure and its stemmata, and as we saw, these stemmata are not linearly ordered. I do suggest that this fact strongly hindered Ibn Sina from finding a notion of scope.

We can say more. Stemmata aren't linearly ordered left-right. But any one branch of a stemma is linearly ordered up-down. To a limited extent one can use this up-down linear ordering to define a notion of scope. It turns out that, at least on one reading of his texts, Ibn Sina does exactly this when he can,



though he never states any general principle behind it. We mentioned two examples above; let me recall them.

The first place is when Ibn Sina has two nested modalities 'on the copula'. In Arabic the modality stated first is the one with greater scope; he seems to attach the meaning of this one below the meaning of the other modality. So if we read the scope of the first modality as everything above it in the branch of the stemma, we correctly have that the second modality is in the scope of the first.

The second place is where he speaks of modality 'on a quantifier'. In practice, as we saw, he understands these modalities as quantifiers over times or situations, with wider scope than the main subject quantifier. So he attaches the modality below the subject quantifier. The same rule as above again correctly gives that the modality has wider scope than the subject quantifier.

In the place where Ibn Sina admits he runs into difficulties, this option is not open to him. The problem is that his time quantifier quantifies a variable that appears in both the subject and the predicate. Ibn Sina always had problems about co-reference between different parts of a sentence, and even more so between different sentences. But here his problem is worse than that, because there is no single branch of the stemma that he can use to compare the two quantifiers.

10. The direction of construction

PTW compositionality says that a certain recursively defined function exists and sets up a certain kind of correlation between syntax and meaning. The recursion is defined in terms of a well-founded partial ordering that is defined syntactically. In logic this partial ordering is usually the partial ordering of the nodes in a stemma that analyses a formula; in linguistics the same usually holds but with a phrase marker in place of a stemma. So in a sense the function is constructed upwards from the bottom of the tree. But this is a mathematical fact, and it says nothing at all about what happens in the mind or the brain to represent the function; as Ibn Sina likes to say, that's a matter for a different discipline.

With Aristotelian compositionality as in Ibn Sina or Frege the position is a little different. This compositionality is supposed to explain a sequence of



events in time: I have a thought, I put it into words, you hear me, you extract my thought from my words. The explanation rests on the idea that the string of words that I use is constructed in parallel with the thought, or later but copying the construction of the thought.

Frege makes some remarks to the effect that we start with the whole thought and carve out parts of it so as to create the structure represented by the *Begriffsschrift*. That suggests that he thinks of the construction as starting at the top of the stemma and working downwards. Some readers have found this puzzling. One possible source of the puzzlement is a confusion with PTW compositionality, which works upwards in the sense we mentioned above.

Apart from pointing out that source of confusion, I would rather not add to this discussion. Frege's remarks might not be intended to describe how the thought is constructed step by step through time. In fact Frege shows rather little interest in mental processes.

With Ibn Sina the position is different. He certainly intends to talk about the process of construction of a sentence meaning. In fact he frequently refers to the process as *taşammur*, 'conceptualisation'. As we saw, he describes the construction of a compound meaning as like the construction of a house. The construction proceeds by attaching further meanings to whatever meaning is already in place. Read literally, that should indicate that the compound meaning is constructed from the root node downwards.

But there are signs that what he has in mind might be more complicated. He often talks of attaching [EVERY] to some other meaning, without any suggestion that that other meaning has already been attached to something higher up. We could extend his building analogy to make sense of this. Sometimes the builder assembles a kitchen cabinet or an oven, and then installs it in the building as a unit. The attachments are as before, except at the point where the cabinet is attached as a whole, not just its backplate. This is one place where we already know that dependency grammar fails to make needed distinctions: there is no way to indicate the difference between attaching a principal subgraph as a whole, and attaching first its head and then the other parts of it.

In the case of constructing a compound proposition out of two clauses, Ibn Sina always speaks as if the two clauses are already available at the stage when



they were attached. Analogy would suggest that he thinks a simple proposition is constructed by first assembling the whole subject meaning and the whole predicate meaning, and then combining these with a copular meaning. But his language in describing the adjunctions added to subject and predicate meanings suggests that these are added to the head in both cases, or to whatever has already been added to the head.

I'm not sure we can usefully take this question any further until we have a better idea of what the answer would imply for Ibn Sina's understanding of semantics. One important question to answer along the way is exactly what he thinks the attachment relation achieves in each case. We do have some information about this. For example Ibn Sina seems to take the view that adjunctions to the subject head meaning normally play the role of modifying that meaning, and the usual form of modification is what he calls 'restriction'. Ibn Sina has quite a large vocabulary for talking about meanings being modified or unmodified, and for different kinds of modification.

11. Top-level processing

Ibn Sina used and taught a procedure for formal logic. For simplicity I take only the case of simple propositions and syllogisms that use simple propositions.

If we are using logic to validate an argument or to make inferences from given premises, the first step is to establish which sentences are the premises. Suppose we have two premises, both of them simple sentences. We must find the subject, predicate and quantifier and an indication of the quality (affirmative or negative). At this stage in the procedure we ignore any modality, and we can ignore all of the subject and predicate except for their head words.

With this much information we can say that if the premises do form the premises of a valid syllogism, it will be in a certain figure. Looking up the rules for that figure, we can tell whether there is in fact a valid syllogistic conclusion, and by what syllogistic mood.

Suppose all is well so far, and we have a conclusion. At this stage we move to what Ibn Sina calls 'taking care of the conditions'. This means restoring the modality if any, together with any further contents of the subject and the



predicate, in order to check whether the conclusion survives when these parts are restored, either as it was before or with appropriate modality or conditions added to it. This is broadly the same sense of 'taking care of the conditions' that was mentioned in section 8 above, except that there the logical operation was forming a contradictory negation while here it's forming a syllogistic conclusion.

At this point Ibn Sina is in trouble, because the set of conditions that might have been attached is open-ended, and we can't hope to write down formal rules that cover all cases. So he does what he can. He adopts, and in some cases aims to correct, Aristotle's rules for the case where the added material is just modalities on the copula. Some other conditions can be treated as like a modality, for example the condition that the predicate holds at all times is like the condition that it holds with necessity. For Ibn Sina, in these cases Aristotle's rules are a clue to how we should proceed, though we need to check them out to see if the results are convincing. Ibn Sina also discusses one or two special inference rules.

He also works out some other cases which involve adding an assumption 'If p ...'. I am assuming these cases are covered by the rubric of 'taking care of the conditions', partly because this kind of assumption is the first thing one thinks of when 'conditions' are mentioned. But there is something unexplained here, since it's not obvious that a condition of this kind is sensibly attached either to the subject or to the predicate.

What Ibn Sina does not do is to ask how Aristotle's logic would need to be reorganised if we were to have any hope of extending it to a broader range of arguments. Ibn Sina's Arabic, Persian and Turkish successors did construct formal logics covering some of the conditions that he had called attention to, though none of them came anywhere near modern first-order logic.

With hindsight we can see one particular barrier that neither Ibn Sina nor his successors in the East succeeded in breaking down, or even in identifying. The barrier is that all of Aristotle's rules operate at the top syntactic level of the sentences involved. Ibn Sina's procedure described above is a good illustration of this: the syllogistic rules operate only on the subject and predicate head terms together with the quantifier and the quality. But it's easy to construct examples of valid inferences where the real work takes place much deeper down inside the sentences. How can one improve the inference



rules so that they dig down deeper into the formulas?

One approach is to paraphrase so as to bring deeper features to the surface of the sentence. Leibniz saw this and made some limited progress in this direction. But one thing that his examples showed was that something more radical is needed.

Actually there are two kinds of rule needed; Frege describes them both. They both involve taking a compound expression and writing it as $F(G)$, say, where G is a part that could be at any depth in the syntactic structure. The first kind of rule (I) is that if $G = G'$ then we can replace $F(G)$ by $F(G')$. Leibniz had this rule implicitly, but the fact that it's independent of the depth of G was first pointed out explicitly by Boole, who adapted the rule from standard mathematical practice.

The second kind of rule (II) is where $F(G)$ and G are sentences, G is not within the scope of any negation in $F(G)$, and we can deduce G' from G . In this case we can also deduce $F(G')$ from $F(G)$. There are two most useful cases of this. The first (IIa) is where $F(G)$ takes the form 'If $p, q \dots$ and r then G' '; in this case we can assume the antecedent, carry out the inference from G to G' and then discharge the assumption. The second (IIb) is where $F(G)$ adds a quantifier prefix to G . Note that it would be enough to have (IIa) with a simple antecedent 'If $p \dots$ ', and (IIb) with a single quantifier in the prefix, because (given some simple book-keeping) we could iterate.

Putting all these cases together allowed Frege to develop in *Begriffsschrift* logical system with the strength of first-order logic and more.

How close did Ibn Sina come to these rules? I don't think he had any thoughts in the direction of (I). There are some small indications that he could handle (IIb), but nothing systematic. But he most definitely did have (IIa). In this way he made one of the most significant advances towards first-order logic before the nineteenth century. The main thing that prevented him from going further was that he always considered logic as about single inference steps, just as grammar is about single sentences. It never occurred to him, or to any logician before the generation of Peano and Frege, to take an entire complex argument and check it formally as a whole.

As it stands, Ibn Sina's procedure for 'taking care of the conditions' doesn't involve taking the conditions in any particular order. (Or if it does, I missed it.) We saw that he lacks the information about the order of attachment that



modern X-bar theory explores. He does mention one formal procedure that could be relevant here. We can take a part of the subject or predicate meaning and amalgamate it into a single chunk, and then introduce a new word for that chunk. The effect would be that given a subject or predicate head meaning with several conditions attached, we could tie some of the conditions into the head so as to create a larger head, and then be free to release the other conditions for 'taking care'.

We shouldn't leave the topic of logical processing without mentioning conversion. For Aristotelian logicians, another basic procedure — alongside finding contradictory negations and syllogistic consequences — is 'converting' a simple proposition by swapping its subject and predicate, and in certain cases changing its quantifier, so long as the result is a consequence of the original proposition. Since this is a logical operation, presumably Ibn Sina sees it as operating on the sentence meaning. This could give us a further source of information about how he sees the grammar of meanings. For example he converts 'Nothing white is black while it is white' to 'Nothing black is white while it is black'. If the predicate in the first proposition was the meaning of 'black while it is white', then the conversion should yield 'Nothing black while it is white is white'. So his conversion indicates that he has some insight into the logical forms of these sentences, as we might put it today. But exactly how he would describe that logical form still eludes us. There are no useful comparisons with Frege here, because conversion is not a logical operation that Frege recognises.

12. Final comparisons

Ibn Sina's grammar of meanings had no direct influence of any kind on Frege's *Begriffsschrift*. Nevertheless the two projects have enough in common to allow us to make detailed comparisons between them. Probably the overlaps have two main sources: the Aristotelian background that Ibn Sina and Frege shared, and the fact that they were coming at the same range of questions about language.

Frege's *Begriffsschrift* unquestionably a more precise, rigorous, well-integrated and innovative system than anything that Ibn Sina has to offer. Frege had important advantages, for example that mathematics had progressed and he



was himself a much better mathematician than Ibn Sina. But probably the decisive difference was that Frege had, right from the start, a narrowly focussed specification of the job that he wanted *Begriffsschrift* do. By contrast, Ibn Sina's grammar of meanings was more an open-ended set of observations without any overarching purpose.

It's illuminating, though historically inaccurate, to think of some of Frege's innovations as repairs for things that go wrong in Ibn Sina's system. But replacing Ibn Sina's grammar of meanings by *Begriffsschrift* 't necessarily be progress on all fronts. In fact Ibn Sina's full and detailed discussions allow us to guess how he might have criticised *Begriffsschrift*. He would probably have found it too far removed from our natural intuitions, and he would have been suspicious of Frege's tendency to insist that only one way of doing things is right. He would also have felt that Frege's justifications are sometimes too metaphysical and sometimes simply inadequate. It could be argued that some developments in the last hundred years support Ibn Sina against Frege on these points.

But ultimately both Ibn Sina and Frege rely on a rather chemical notion of meanings, that allows us to split meanings into their elements and build them up into compound structures by putting bonds between them. Today I think few people have any confidence that this is anything more than a handy set of metaphors. We have moved on. But we can look back on both systems — Ibn Sina's and Frege's — as major achievements in the history of ideas, and as challenges to our understanding.

Bibliography

- [1] Wilfrid Hodges, 'The grammar of meanings in Ibn Sina and Frege', at <http://wilfridhodes.co.uk/arabic25.pdf>.
- [2] Wilfrid Hodges, 'Notes on the history of scope', submitted for a Festschrift. Publication details will be put in [1] when available.